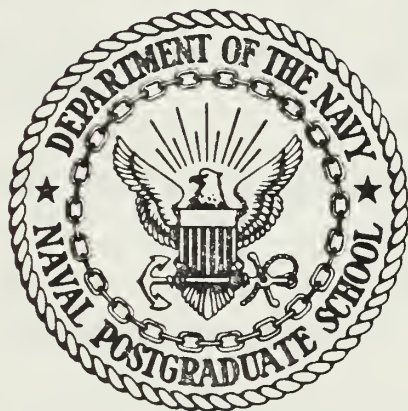


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THESIS

NETWORK REPRESENTATION
FOR COMBAT MODELS

by

Thomas P. Krupenevich

December 1984

Thesis Advisor:

James K. Hartman

Approved for public release; distribution unlimited

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Network Representation for Combat Models

by

Thomas P. Krupenevich
Major, United States Army
B.S., University of Connecticut, 1973

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
December 1984

ABSTRACT

A general network methodology for combat processes is presented in this thesis for use in the Airland Research Model. Specifically, two processes are developed in detail: the underlying transportation system and the command and control connectivity structure. Attributes necessary to support representation of each system as a network model have been identified.

TABLE OF CONTENTS

I.	THE AIRLAND RESEARCH MODEL	9
A.	BACKGROUND	9
	1. Purpose of the Research Model	9
	2. Methodology Research Areas	10
B.	PURPOSE AND GOALS OF THIS REPORT	10
	1. Purpose	11
	2. Goals	12
C.	ASSUMPTIONS OF THIS REPORT	13
D.	CURRENT TERRAIN MODELLING TECHNIQUES	15
	1. Hex Terrain	16
	2. Digitized Terrain	17
	3. Functional Terrain	18
E.	CAPABILITIES OF A NETWORK REPRESENTATION	18
F.	ONGOING RESEARCH WORK	19
II.	GENERAL NETWORK CONCEPTS	21
A.	DEFINITIONS	21
	1. Pieces of the Network	21
	2. Attributes	22
	3. Structures within a Network	23
B.	UTILIZATION OF NETWORKS	24
	1. Network Representations	24
	2. Network Algorithms	28
C.	NETWORK REPRESENTATION WITHIN THE AIRLAND RESEARCH MODEL	32
	1. Transportation Network	32
	2. Command and Control Structures	33
III.	DEVELOPMENT OF THE TRANSPORTATION NETWORK	36

A.	INTRODUCTION	36
B.	NOTATION	37
C.	THE EXECUTION NETWORK	37
1.	Arcs: Uses and Attributes	39
2.	Nodes: Uses and Attributes	46
D.	PLANNING NETWORKS	52
1.	Extraction of Planning Networks from the Execution Network	52
2.	Attribute Coding Requirements	53
E.	UNIT REPRESENTATION ON THE TRANSPORTATION NETWORK	55
1.	Necessary Unit Information	55
2.	Representing a Unit on the Planning Network	58
3.	Representing A Unit on the Execution Network	59
4.	Multiple Units on the Execution Network	61
F.	OBSTACLE REPRESENTATION ON THE TRANSPORTATION NETWORK	62
1.	Needed Obstacle Information	63
2.	Obstacle Modelling Methods	64
3.	Measuring Obstacle Effects	69
G.	MODELLING FLOW ON THE PLANNING NETWORK	71
1.	Use of Homogeneous Flow Models	71
2.	Minimum Cost Route Selection	77
3.	Selection of a Unit's Future Location	81
H.	DISCRETE SIMULATION OF THE EXECUTION NETWORK	84
IV.	DEVELOPMENT OF THE COMMAND AND CONTROL CONNECTIVITY NETWORK	90
A.	BACKGROUND	90
B.	INFORMATION AND TASK ORGANIZATION	93

1.	Types of Information	93
2.	Effects of Task Organization	94
C.	FORMULATION OF THE C ² NETWORK	103
1.	Subscript Notation	103
2.	Resolution Issues	104
3.	Data Base Attributes	106
4.	Current State Attributes	108
5.	Discrete Event Simulations	109
V.	CONCLUSIONS AND RECOMMENDATIONS	113
A.	CONCLUSIONS	113
B.	COMPARISON OF FLOW MODELS AND DISCRETE SIMULATIONS	114
1.	Use of Continuous Flow Models	115
2.	Use of Discrete Network Simulations	116
C.	RECOMMENDATIONS	116
	APPENDIX A: OPERATIONAL TERMS AND DEFINITIONS	118
	LIST OF REFERENCES	121
	INITIAL DISTRIBUTION LIST	122

LIST OF TABLES

I	Support Capacity of Arc Surfaces	42
II	Summary of Arc Attributes	45
III	Node Classifications	48
IV	Summary of Node Attributes	51
V	Typical Unit Allocation Scheme	95
VI	Command Relationship Information Flow	97
VII	Summary of Message Category-Type Combinations .	104
VIII	Summary of Modes of Transmission	104

LIST OF FIGURES

1.1	CORDIVEM Terrain Features	17
2.1	Tree Networks	24
3.1	Arc Representation of an Obstacle	65
3.2	Arc Representation of an Obstacle at a Node	69
4.1	Air Defense Artillery Support Relationships	98
4.2	Artillery Support Relationships	100
4.3	Aviation Command and Support Relationships	101
4.4	Engineer Command and Support Relationships	102
4.5	Discrete Event Simulation Example	110

I. THE AIRLAND RESEARCH MODEL

A. BACKGROUND

Current doctrine for United States ground forces advocates fighting the airland battle to defeat enemy forces. The airland battle requires complex integration of fire support weapons, Army and Air Force aviation assets, and ground maneuver forces. Doctrine calls for the enemy to be defeated by first stopping his forward elements, second, by defeating his reserve echelons before their combat value can influence the battle, and third, by attacking his resources to prevent reconstitution of previously defeated forces. The goal of the Airland Research Model is to simulate this doctrine for study and evaluation.

1. Purpose of the Research Model

The research model has three primary purposes:
[Ref. 1: p. 1]

- The development of modelling methodology appropriate for the very large scale but sparsely populated rear areas involved in the (non-FLOT) interdiction battle and for the command and control of the airland battle force. The FLOT (Front line of Own Troops) is defined in Appendix A.
- The application of these methodologies in the construction of a simulation/wargaming model initially focusing on two-sided interdiction.
- The (eventual) use of the model to perform research on the conduct of the total airland battle.

2. Methodology Research Areas

Five areas of modelling methodologies have been identified for use in the research model. These areas are summarized below and further explained in [Ref. 1].

- Develop a model that can be operated in either a closed or man-in-the-loop fashion
- Develop rule based systems to represent command and control and related processes
- Develop a generalized network methodology
- Develop a generalized value system applicable to all essential combat features found in the research simulation
- Develop methodology for a variable resolution architecture to include methods for aggregation and disaggregation as dictated by functions, mission and situation.

B. PURPOSE AND GOALS OF THIS REPORT

This report will begin to develop one specific research area, the development of a generalized network methodology. Early work on the research model stated that the major features of the simulated systems and their environment were to be represented in one or more of the following generalized coordinate systems [Ref. 1: p. 5].

- Hierarchical Army unit organization space.

The unit organization space represents the superior-subordinate system of organic unit relationships from which 'sister' units subordinate to the same superior unit can be derived. This structure would remain mostly static throughout the battle.

- Combat task force organization space

The task force organization space represents the operational relationships that occur due to cross-attachment of units; this network must allow frequent changes in unit relationships.

- Communications interconnectivity network
- Transportation interconnectivity network

Both of the above networks are flow networks in that they represent flows, capacities and delays along connecting links. Methods to represent network destruction and reconstitution will be addressed.

- Geometric location in space and time

The problem of locating and moving units must be addressed in order for planning modules to make logical transitions from one state to the next.

1. Purpose

The purpose of this report is to define a general structure for each of the networks which will be required in the research model. In addition, this report will outline conditions necessary for the use of each network, representation problems particular to each network which require further development, and identify attributes of the network for which data collection may begin.

The major effort of this report is focused on two general networks: the physical transportation network and the command and control (C²) interconnectivity network. The physical network will be the underlying structure of an 'execution' network used to model the current state of the network and the location of combat units. In turn, 'planning' networks will be an abstraction of the execution network and used to predict optimal unit movements in terms of maximum flow and shortest paths. The development of these three networks addresses problems related to the five coordinate systems just cited. The C² network to be developed will be used to identify a combat unit's hierarchical organization space, its combat task force space, and the required communications interconnectivity between units. The transportation interconnectivity network and geographic

locations on that network will be represented via the planning and execution networks to be developed.

2. Goals

The underlying goal of this thesis is to demonstrate the applicability of a network representation of a battlefield in regards to terrain representation and organization of the command and control system. In particular, each of the following goals will guide the development of the transportation and C² models.

- Develop a network model of a battlefield which results in an economy of representation that is not found in other battlefield models. A network representation can be used to represent features of the environment and terrain necessary to the interdiction battle. A tree-like network structure can be used to represent only desired headquarters and staff sections.
- Develop a network which permits the use of varying levels of resolution at different parts of the battlefield. In a transportation network, the level of resolution can be controlled by varying the number of locations (i.e. nodes) represented in the network and the routes which connect those locations (i.e. arcs). In a C² network, the level of resolution can be controlled by varying the number of units or staff sections, (nodes), and the means of communications between two units, (arcs).
- Develop a network representation of the battlefield which will exploit the large number of currently existing network algorithms.
- Develop a network which provides a structure that can be used to simulate the movement of multiple units, network congestion, and network interdiction and repair.

- Develop a set of arc and node attributes which support both continuous flow and discrete simulation modelling.

Finally, this thesis will compare the advantages and disadvantages of using network flow models and discrete simulations and draw conclusions as to the appropriateness of each model. This discussion will be found in Chapter V.

C. ASSUMPTIONS OF THIS REPORT

It is necessary to assume that certain trends in ongoing research will continue to be developed in support of the research model. These assumptions are based on objectives outlined in initial position papers concerning the research model, [Ref. 1], and by additional work supporting the unit value system, [Ref. 2].

First, a U.S. corps level model is being developed. Therefore, the model must represent the opposing forces deployed against this corps on a terrain model large enough to support the movement of combat and support units. This thesis will be developed assuming the U.S. corps is in a defensive posture and desires to disrupt the movement of Soviet forces to the FLOT. The final model will support both U.S. offensive and defensive operations.

Second, a value system exists with which one can compute a unit's present and future value. Preliminary work on the generalized value system is outlined in [Ref. 2].

In brief, each maneuver or fire support unit in contact has a 'Basic Inherent Value' which is based on its current state. The values of combat support (CS) and combat service support (CSS) units are derived from the increase (or decrease) they provide in the inherent value of a combat unit. Units not in contact and possessing unused support will be treated as an asset that will mature in time, i.e. their current value is a discounted version of their future worth.

Third, the initial research model will not represent the FLOT battle in detail. As stated, one of the primary purposes of the research model is to model interdiction in the sparsely populated rear area. The FLOT battle will eventually be represented in the model however, and the network formulation must support its (the FLOT battle) integration into the model. This thesis will only address attrition caused by obstacles or indirect fire. It must be understood that the inclusion of direct fire attrition considerations may result in routings and movement schemes that at first seem infeasible or sub-optimal.

Fourth, decision and planning algorithms will be developed at the battalion level and above for the combat forces. Company and battery sized maneuver forces will be represented in the model only to execute the results of the battalion decision modules. Research work in CS/CSS functions will define the lowest levels necessary for decision and planning. References throughout the thesis to unit hierarchies or echelons will be referring to the normal army chain of command in which authority increases from company to battalion, to brigade, division, and corps. The terms hierarchy and echelon will be used interchangeably.

Fifth, the research model, in its final form will be composed of numerous modules, many of which still need to be identified and developed. In this report, a module is understood to be a subroutine pertaining to a specific function found within the research model. The modules developed for each function will address decision making, planning future operations, and executing current missions. A unit processor is understood to be an algorithm for decision making and planning which is developed for a specific headquarters or staff section. It must be understood that details of these processors have not been developed at this time. Thus, this thesis can only talk in general terms

about the relationships among the processors and the network models.

D. CURRENT TERRAIN MODELLING TECHNIQUES

Representing combat functions as a system of networks is a departure from the current practice of explicit terrain representation. Current modelling techniques require large data bases to represent numerous aspects of terrain such as slope, elevation, forestation, and cities. The network approach is an attempt to represent the terrain, primarily through use of the transportation system, as an abstraction of its actual state. The topology characteristics required by explicit terrain models will be reflected in the arc and node attributes.

The research model will gain an economy of representation of the topology because it has the flexibility to represent only those terrain features applicable to a stated level of resolution. The corps rear is sparsely populated with combat and support units relative to the overall size of the corps sector. Furthermore, large portions of the corps sector will never support unit movement or facilities and have no reason to be modelled. A network representation allows one to select only those features which, because of their nature, give the controlling force a marked advantage over the other force (e.g. key terrain). Additionally, those roads or cross-country arcs which can either expedite or delay movement can be identified and modelled.

In a corps level model, direct fire combat requires observation, detection, and lines of sight along a narrow strip of terrain relative to the overall size of the sectors of the two forces. Existing methods of terrain representation may still be required to model the details of such combat. The majority of units moving in a corps sector,

especially the CS/CSS assets, do so without receiving direct fire. The movement of the units can be modelled via a network abstraction of the transportation system. Numerous algorithms exist which allow one to exploit the features and structure of such a mathematical model.

Many of the terrain modelling techniques in current use restrict the model to one level of resolution and force the terrain data base to store attributes for large sections of the battlefield which are not used. To put current terrain modelling procedures in perspective, the following is a short discussion of three terrain modelling methods currently in use: hex terrain (used in CORDIVEM), digital terrain (DYNTACS), and functional terrain (STAR).

1. Hex Terrain

The Corps-Division Evaluation Model (CORDIVEM) is a corps level model in which the terrain is represented as a series of hexagons, (hexes), each approximately 3.5 kilometers in diameter. The terrain underlying each hex is characterized by a three digit array which represents urbanization, forestation, and roughness. Each individual attribute is an average of the Defense Mapping Agency's data base of points 12.5 meters apart which underlie each CORDIVEM hex. The attribute based on the resulting average is then assigned a code in CORDIVEM and assumed homogeneous throughout the hex. Figure 1.1 illustrates the codes for each hex attribute.

The geometry of the hex is used to represent unit movement, roads, rivers and obstacles. Unit movement and a road system, if one exists, are modelled from the center of one hex to the center of an adjacent hex. Movement on a cross-country route is penalized by reducing a unit's allowable speed. Rivers are modelled along the edge of a hex, and coded as indicated in figure 1.1. An implicit bridge is

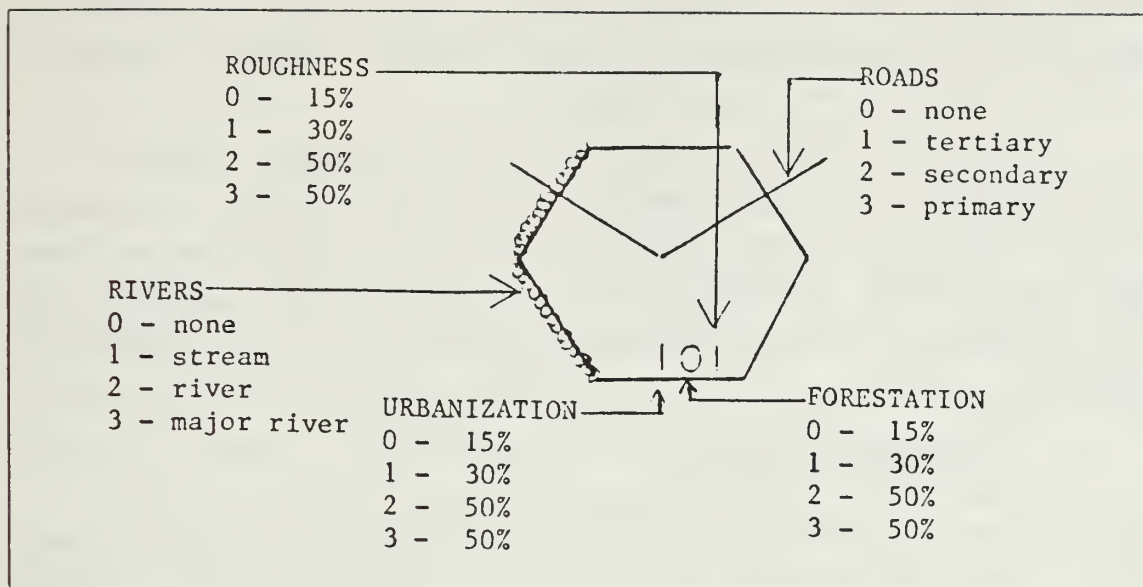


Figure 1.1 CORDIVEM Terrain Features

assumed at the intersection of all roads and rivers. The bridge can be explicitly modelled if it may affect movement factors or possibly be destroyed. An obstacle occurs on the edge of the hex and is explicitly inserted into or removed from the model. Only completed obstacles are inserted during the execution of the model; the dynamic construction and reduction of obstacles are not modelled.

2. Digitized Terrain

The Dynamic Tactical Simulation (DYNTACS) model is a two-sided, dynamic Monte Carlo simulation capable of modelling the combat process from the individual crew to battalion engagements which utilizes a concept referred to as digitized terrain [Ref. 3:p. 9]. Macro terrain in DYNTACS is usually represented as 100 meter squares with attributes for the corners being supplied by data available from the Waterways Experiment Station. These corner attributes provide information about such features as elevation,

forestation, and location. Each square is then divided diagonally, resulting in a series of equal sized triangles, varying in slope and orientation.

This approach assumes that the terrain modelled by each triangle is homogeneous and that elevation changes between end points are linear. Because the grid has uniform spacings, sudden changes in the terrain or its surface are not detected; instead they are represented with geometric overlays.

3. Functional Terrain

The Simulation of Tactical Alternative Responses (STAR) model is an air-ground simulation that models individual engagements up to the brigade level. The STAR model reduces the extensive computer storage required by digitized terrain models by representing the terrain as a series of continuous terrain surface functions and storing only the parameters of the functions representative of each hill. Terrain in the STAR model is continuous and requires no interpolation of elevation values.

Characteristics of the terrain which affect movement, detection and line of sight such as forests and cities are represented as a series of overlays which are modelled in conjunction with the appropriate terrain location. A detailed explanation of functional terrain is available in [Ref. 4].

E. CAPABILITIES OF A NETWORK REPRESENTATION

The execution, planning, and C² networks have been briefly introduced, and will be discussed in greater detail in following chapters. For now, it is sufficient to note that the execution network will be a high resolution representation of the area being modelled and used to simulate

the movement and location of combat units. The execution network will be the data base equivalent to explicit terrain representation.

Planning networks will be abstractions of the execution network and are concerned primarily with optimizing network flow of future operations. Network abstractions of real systems have value in planning because of the mathematical structure inherent in the network model. Continuous flow models on the network can be used for aggregated representations of combat unit movement or logistic support.

Each network can also be interdicted by the insertion of nodes and arcs representative of obstacles at selected points in the network.

A network model can serve as the framework in which the C² doctrine, and tactics of two opposing forces are represented. Doctrine generally states how a force should be organized to respond to a certain mission or situation. Connectivity models can be used to portray both hierarchical and operational associations. Depending on the level of resolution of the C² model, and the objective of the study, information transmission and processing may also be modelled using the network structure.

F. ONGOING RESEARCH WORK

The research model is being developed in part through the thesis work of students at the Naval Postgraduate School. Previous works have discussed the communications connectivity model in detail, [Ref. 5], and several aspects of the Soviet force structure to include the development of a Soviet planning model and a Soviet logistics model. Current student research is focused on the U.S. logistics and maintenance structure and the employment of engineer assets to support missions of mobility, countermobility, and survivability.

In this report Chapter II discusses general network concepts, systems which can be represented as networks, and introduces the execution, planning and C² networks. Chapter III presents the development of the execution and planning networks. Several properties of an individual unit moving in the transportation network are discussed. Maximum flow and shortest path problem formulations using the attributes presented in Chapter III demonstrate the applicability of network representations for planning future combat operations. Chapter IV presents the development of a command and control connectivity network; information and message flow are discussed as they pertain to the C² network. This chapter does not develop an algorithm for processing information within a unit; however it identifies several of the actions which do occur when information is received at a unit. Chapter V presents conclusions and topics for further research suggested by this work.

II. GENERAL NETWORK CONCEPTS

The value of a network lies in its ability to represent a real system such as a transportation system or scheduling problem as an abstraction of the environment in which it is found. Wagner states that "the key justification (for studying networks in great detail) is that the mathematical characteristics of network models are so special that by exploiting these structural properties you can obtain major efficiencies in finding optimal solutions", [Ref. 6: p. 169].

Sections A and B of this chapter present general network definitions and theory, and algorithms which exploit network structure. Section C introduces the specific networks which will be used in the research model.

A. DEFINITIONS

1. Pieces of the Network

Real systems have certain common characteristics which form the basis of network models. This section will be concerned with two of these common elements, nodes and arcs, as explained by Price, [Ref. 7: p. 5].

Nodes are fundamental locations in the network; they can represent geographic locations or levels in a hierarchical organization structure. Nodes exist to represent sources, sinks or routing points in the system being modelled.

Arcs are ordered pairs of nodes which represent connections between nodes within the real system. Arcs are used to represent the routes a commodity can take while traveling in the network. If arc (x,y) is equal to arc

(y,x) in all attributes, then the arc is said to be 'undirected'; otherwise, the arc is 'directed'. The existence of arc (x,y) but not arc (y,x) implies that arc (x,y) is directed, with travel allowable only from node x to node y. Arcs may represent sequencing in planning networks; arc (x,y) implies that event x occurs prior to event y. Arcs are also used to represent relationships in hierarchies; arc (x,y) may mean that x is y's superior.

2. Attributes

The attributes of an arc or node are arrays of information that describe pertinent information about that arc or node. An attribute is assumed to have uniform representation along the entire arc or node. Several attributes listed below will be referred to extensively throughout this report:

- Flow
- Capacity
- Length
- Cost

Flow of a commodity through a network, measured as an amount per specific time interval, occurs along arcs and through nodes. Flow along an arc is assumed uniform. Flow through a node is represented by the sum of all arc flow rates terminating at that node.

Capacity represents the maximum allowable flow rate along an arc or through a node. Capacity may be measured in units identical to the flow rate.

The length of an arc is the true distance of the route represented by that arc. This distance may be greater than the Euclidean distance determined by the geographic locations of the nodes. For example, an arc may represent a curved road which is longer than the straight line distance between its two end points.

Cost is a non-negative penalty associated with an arc (node), incurred as a result of using that specific arc (node). Costs vary with the system being represented; time and distance are common costs. Terms that convey a similar idea such as length and distance are often used interchangeably.

3. Structures within a Network

Network pieces become relevant when several 'adjacent' arcs and nodes are linked together. Two nodes are said to be adjacent if they represent the two end points of a single arc; two arcs are adjacent if they have a common node, [Ref. 7: p. 9]. A collection of adjacent nodes and arcs together forms three structures which will be of interest here: chains, paths, and trees.

A chain is a set of at least two arcs such that:

- There are exactly two arcs of the set which are each adjacent to exactly one other arc of the set,
- All other arcs of the set are each adjacent to exactly two other arcs of the set [Ref. 7: p. 12].

A path from any node i to any node j is a chain of directed arcs beginning at node i and terminating at node j . All arcs are aligned to allow travel from node i to node j .

Within a connected network, a tree is a connected partial subnetwork which contains no cycles. A network is connected if, from each node i there is at least one chain terminating at every other node j ; a cycle exists if, beginning at some node i , a chain returns to that beginning node. A spanning tree is a tree that connects all the nodes of the original network. Trees are useful in representing organization structures; a tree and a spanning tree are shown as figure 2.1.

from
[Ref. 7: p. 16]

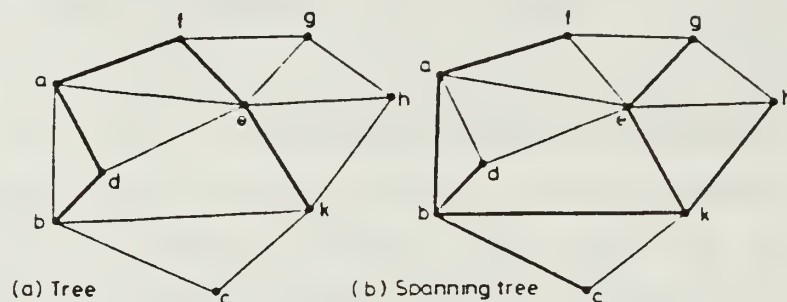


Figure 2.1 Tree Networks

B. UTILIZATION OF NETWORKS

1. Network Representations

As previously stated, network methodology allows one to represent a system as an abstraction by taking advantage of the common features found within the system. Network models are often used to represent the following:

- Transportation flow networks
- Discrete network simulations
- Connectivity networks
- Activity networks

a. Transportation Networks

Transportation networks are used extensively to model product distribution. In general, there exists a need for one or more products which are located at points other than the demand site. The following properties exist within a transportation network.

Nodes represent a location in a geographic coordinate system. Nodes act as supply points, demand sites, or simple junction points. Examples include physical terrain features such as road junctions, facilities such as manufacturing and demand sites, and transshipment centers, where items could possibly change modes of transportation.

An arc represents the existing route over which the product flows. When linked together, the arcs virtually form a road map of the existing transportation system. Examples of arcs include roads, railways, pipelines, and air and sea links. A transportation network need not be restricted to one single type of arc; it can represent simultaneously several modes of travel for product distribution.

Flow in a transportation network is usually a measurable quantity of product over some time interval. The flow can be either a single product (single commodity) or several different products moving at once (multi-commodity).

The capacity of an arc or a node is the largest instantaneous flow that an arc or node can support.

The length of an arc is the actual 'road' distance that must be travelled from end node to end node. This may be greater than the Euclidean distance, as explained earlier.

The cost associated with an arc or node may take one of several forms; besides length, for example, cost could be measured in time requirements or attrition.

b. Discrete Network Simulations

Discrete network simulations utilize the structure of a transportation network, however there are several advantages to using a simulation when a high resolution study is being conducted.

First, a simulation allows one to represent transient flow versus the steady state flow modelled on a transportation network. The optimal solution for a transportation network results in a continuous flow for the commodity. A simulation, however, allows one to move discrete packets of goods around the network, as either unique requirements or as recurring demands. The use of a discrete packet also allows for the simulation of delay time at a node and comparison of alternate routing schemes. Simulations, though, only return a feasible and not necessarily optimal solution to a problem.

Second, simulations can reflect transient changes to the underlying network structure. With every change to the network in a steady state flow model, the flow rates must be re-optimized. There is usually no natural transition from the network state prior to the change to the state recommended by the re-optimization. With a simulation, however, the network begins its transition with the current state of product flow around the network, and provides for a logical transition to the next state. This is extremely valuable when used in planning modules to evaluate several future courses of action.

A disadvantage of using a discrete simulation is that algorithms must be developed for each situation. (References in this report to a discrete simulation algorithm assume that such an algorithm to support the research model will be developed in the future). Despite the difficulty in constructing discrete simulations, many such models have been built and are currently in use; one such example is the LOGATTACK model which focuses on network representations in a restrictive area to simulate movement of logistic products.

c. Connectivity Networks

Connectivity networks are used to represent associations as they normally occur; flow of a commodity in this network is not mandatory. The purpose of such a network then becomes twofold:

First, a network identifies relationships between 'entities'. Entities can represent individual items to entire organizations; examples can be found in family trees, levels in an organization hierarchy, and lateral relationships within one level of a hierarchy.

Second, connectivity networks can also represent paths over which information flows in some form, as either transient or steady state flow.

Arcs in a connectivity network are used to denote the link between the source and the destination of a communication. Common terms for this relationship include the parent-child, commander-subordinate, and predecessor-successor relationships. The direction of the arc often implies the type of transmission which is allowed between the two nodes.

Nodes in a connectivity network denote the relative location of the entity within the network. In a family tree, for example, a node denotes the identity of an individual. In an information network, a node indicates either the sender or receiver of a communication.

d. Activity Networks

Many problems in sequencing and scheduling can be looked upon as problems in network theory, [Ref. 7: p. 2]. A network representation links together several individual jobs or processes that make an entire sequence.

In scheduling networks, a node represents both completion of all processes entering that node, and the beginning of all processes exiting that node.

Arcs represent the activities, i.e. jobs or collection of jobs that occur as time progresses. Arcs are directed and define the sequential connection of processes. Job progress is measured as either completed, in progress or waiting to start.

Time is used to represent cost if one is concerned with a deadline, or just wishes to compare actual versus planned time requirements.

There are specific arc features that must be present in a scheduling network. The arcs must represent the logical sequence in which the jobs are completed. An activity beginning at node j cannot begin until all processes entering node j have been completed. Dummy activity arcs, requiring zero time penalty can be utilized to show proper sequencing of events.

Scheduling networks are completely directed and have no cycles. It is not feasible for the beginning of any event already completed to depend on an event that has not yet begun. Additionally, scheduling networks must begin and end at unique nodes. If necessary, this can be accomplished by creating an initial (terminal) node with dummy arcs to the succeeding (preceding) nodes.

2. Network Algorithms

The ability to represent a complex system as a network would be meaningless unless some advantage was gained by that representation. In fact, there exist several algorithms that take advantage of network structure to provide information about the system being modelled.

Four classes of algorithms are listed below; each class will be discussed in turn.

- Path determination
- Flow optimization
- Location results

- Activity scheduling

- a. Path Determination

Path determination algorithms are used for moving about the network when both the starting and terminating nodes are known; the resultant path satisfies the desired objective. Objective functions can be expressed in time, distance or any other 'cost' appropriate to the network. Algorithms can be found in Wagner, [Ref. 6: ch 7] and Price, [Ref. 7: ch 3] that solve for the following paths:

- Shortest (longest) path through a network,
- Kth shortest path through a network, and
- All paths through a network.

Shortest path algorithms find the least cost path through a network. Kth shortest path algorithms are useful given the non-availability of an arc or node on the shortest path. Path determination algorithms are used extensively in transportation networks to provide optimal routings and allocation of resources.

- b. Flow Optimization

Flow networks represent a system through which some product flows continuously over time. The two algorithms of interest here are maximum flow and minimum cost.

The maximum flow algorithm determines the greatest amount of product that can be routed from source to sink, given the capacities of the arcs and nodes [Ref. 6: p. 953].

Minimum cost algorithms can be applied two ways on flow networks,

- as a comparison of multiple feasible solutions to maximum flow problems

- as a determination of the minimum cost required to satisfy demands at some terminal node.

Flow algorithms are used extensively in transportation networks. Algorithms exist that optimize either homogeneous or multi-commodity flows. As the complexity of the flow increases, however, so does the difficulty of finding an optimal solution.

c. Location Results

Thus far, algorithms have been discussed which are useful for determining desired paths or flow rates on networks. These algorithms assume that source and sink nodes, supplies, and demands are known prior to computing a feasible solution. Location algorithms are used to determine optimal locations for source or sink nodes being added to an existing network; two concepts for location, as described by Handler, will be introduced. [Ref. 8].

Minimax location algorithms determine a location which minimizes the maximum (path) distance from that location to a potential demand site anywhere on the network. This method considers only potential farthest points and is not concerned with costs to other, closer points.

Minisum location algorithms determine a location which minimizes the average cost of travel to any other point on the network. Cost is expressed in terms appropriate to the network.

Location algorithms are very adaptable to a variety of objective functions and changing requirements. For example, the objective function can be stated as a minisum problem while constrained by minimax requirements. Certain algorithms find solutions only at nodes while others consider all possible parts of the network. Location algorithms can also solve for single or multiple locations. Inverse center algorithms determine the minimum number of

facilities and the locations at which the network should be interrupted in order to satisfy upper and lower distance constraints.

Location methods are applicable to transportation problems and connectivity networks. Within transportation networks, location methods provide facility locations based either on minisum or minimax procedures. In connectivity networks, methods can be utilized for locating communications centers which minimize transmission costs (expressed in time or number of transmissions).

d. Activity Scheduling

Activity scheduling algorithms deal primarily with sequential events, and therefore are applicable to the activity networks discussed earlier. In particular, scheduling networks are the basis of several techniques such as the Program Evaluation Review Technique (PERT) and the Critical Path Method (CPM).

The desired information from such a calculation is usually the longest (critical) path which is also the shortest time in which a project can be completed. Network calculations also identify time reserves that exist along non-critical arcs in the form of early and late completion times.

By proper reallocation of the resources and manpower committed to processes which have time reserves, the critical path may be shortened. Alternately, these resources can be retained and committed only when delays occur that would otherwise cause the critical path to become infeasible.

After calculating an activity network, the arcs can be projected onto a single time line, which then allows a planner to compare ongoing work with scheduled processes.

A detailed presentation of longest path algorithms can be found in Wagner [Ref. 6: p. 233]. and Price, [Ref. 7: p. 87] Additionally Skachko presents algorithms for calculating PERT networks with several examples [Ref. 10].

C. NETWORK REPRESENTATION WITHIN THE AIRLAND RESEARCH MODEL

As stated in the first chapter, the purpose of this report is to develop the structures necessary to model the following networks:

- the underlying transportation system used to support the execution network and the planning networks
- the hierarchical command and control structure.

These networks will be introduced here and developed fully as following chapters.

1. Transportation Network

The transportation network is a complex structure which must be capable of modelling the simultaneous movement of combat and combat support units utilizing various modes of transportation.

a. Representation of the Transportation Network

The features and attributes of networks, generally defined in earlier sections, will be specifically defined in this transportation network as follows:

- Nodes will represent a specific geographic location.
- Arcs will represent a movement artery on which selected combat units or commodities can travel.
- Flow will represent the movement of vehicles and units along paths in the network.
- Capacity will represent the maximum number of vehicles allowed to travel on an arc or through a node per hour.

- The cost of using an arc will usually be measured in time; distance or attrition may be used depending on the tactical situation.

b. Applicable Algorithms

Algorithms which will provide useful information about the planning network are:

- Path determinations (shortest, second shortest)
- Location methods (future unit location)
- Flow algorithms

Discrete network simulations will be used to model movement in the execution network.

c. Examples of Systems to be Modelled

Systems or combat processes that will be represented by this network include:

- Movement of units
- Movement of support items, i.e. fuel, ammunition, repair parts
- Identification of key terrain for use as defensive positions or offensive objectives.
- The rear area interdiction process.

2. Command and Control Structures

The hierarchical command and control structure of military organizations can be represented as a very complicated connected network. This network in turn can be decomposed into several subnetworks, each representing one specific aspect of command and control. This structure is complicated by the practice of cross-attaching units at various levels throughout the hierarchy and the designation to subordinate units of various command relationships and tactical missions, each with a different set of rules for information flow.

a. Representation in the Network

Flow in this network depends on the purpose of the subnetwork. In a subnetwork designed to model information exchange, flow will be the actual communications being passed between units. Nothing will flow in a network which merely represents a parent-child or child-twin relationship between units.

Nodes will represent either a unit or staff agency; a node is the origin or destination of a communication of some sort.

An arc will be a directed link between two nodes indicating the sender or receiver of information. Arcs will also be categorized to restrict the allowable type of information which can flow on them.

b. Applicable Algorithms

Algorithms which will provide useful information about the command and control structure and message flow are:

- Path determination (shortest message path)
- Discrete flow simulations

c. Examples of Information to be Modelled

The assignment of directed pointers between units in this network depends on the tactical mission and manner of control; however, the following types of command and logistics information flow are common to all organizations:

- Orders- specific instructions from superior to subordinate units
- Requests- a course of action suggested by a subordinate unit requiring approval from that unit's immediate superior.

- Situation reports- reports flowing in both directions concerning friendly units.
- Intelligence- information flowing in both directions concerning enemy units.

III. DEVELOPMENT OF THE TRANSPORTATION NETWORK

A. INTRODUCTION

The purpose of this chapter is to develop the data base requirements of the execution network, discuss its use in the research model, and develop the planning networks as abstractions of the execution network. Chapter II presented network components in general terms and introduced the transportation network; this chapter will state exactly what features nodes and arcs represent. Additionally, it spells out the arc and node attributes necessary to quantify the general attributes of 'flow', 'capacity', and 'cost' listed in Chapter II.

The discussion of execution and planning networks associated with different levels of the combat force gives the initial impression of numerous networks storing the same information several times over. However, the nature of military operations restricts a unit's actions to a certain sector; sectors increase in size relative to a unit's location in the hierarchical force structure. Therefore, references to a detailed battalion planning network are referring to a very small portion of the overall corps network in the data base.

The conclusions of this chapter will discuss the applicability of the execution and planning networks as compared to the goals of the thesis stated in Chapter I. This chapter does not develop a detailed algorithm for simulating the movement of multiple units around the network. However, several formulations demonstrate the use of network algorithms. In particular, the conclusions will address:

- The use of homogeneous flow models

- Interdicting maximum flow
- Minimum cost route selection
- Interdicting the shortest path
- Selecting a unit's future location
- Interdicting units on the network

In order to properly develop these conclusions, however, this chapter must first present the attributes of nodes and arcs, a discussion of unit representation, obstacle representation and their effects on the network.

B. NOTATION

The following notation scheme will be used in this chapter unless specifically stated otherwise.

- An arc (x,y) is represented as an ordered pair of end nodes; any arc attribute subscripted by arc notation is referring strictly to that arc.
- A node (z) is represented by a single number or lower case letter enclosed in parentheses; node attributes subscripted by a node refer specifically to that node.
- A path (A) will be represented by a single upper case letter enclosed in parentheses.
- The lower case letter k refers specifically to the kth combat unit on the execution network.

C. THE EXECUTION NETWORK

It must be understood that there will be only one execution network within the research model. This network will model the movement of all combat forces at their lowest level of representation during the execution stage of the model.

This thesis introduces the concept that nodes may have attributes of length and time. This is done for two reasons. First, as higher levels in the hierarchy are

represented, less detail is needed to describe certain features. Second, and more important, the interior of the node is considered homogeneous and reflects a difference between the attributes of the node and the attributes of the arcs incident to that node. For example, assume the lead elements of a combat unit have entered a town, represented as a node with length (diameter) of 5 kilometers, while the remainder of the unit is on an open highway (arc) leading into the town when a tactical nuclear weapon, which can affect both the city and the highway, is detonated. Besides the obvious dependence on range from the impact point, casualties to the unit also depend on damage inflicted by the weapon's blast and heat effects. The damage due to rubble and blowdown will be significant to the portion of the unit in the town and may be negligible to the remainder of the unit. Conversely, that portion of the unit on the open highway will be severely affected by the heat of the blast, while the portion inside the town will be shielded to some extent.

Depending on the size and complexity of the feature represented by the node, it may also reflect a significant economy of representation within the network model. A single node can represent the characteristics of a town in one attribute array. It may be unnecessary to represent the complex street system of that town as numerous arcs and nodes, all carrying the same characteristics while measuring weapons effects. Additionally, higher echelon (corp and division) execution networks are interested more in the connectivity and time delays associated with a node than an actual path through the node.

The pieces of the execution network, (nodes and arcs), describe locations and routes respectively, and together form an abstraction of the terrain being modelled. The attributes of both nodes and arcs must contain the

information required to model the movement of combat units over the network. They must also support the research model goal of using variable resolution at different levels of execution.

The attribute arrays cannot be so long that the efficiency of a network representation is lost. The attributes presented here reflect the minimum amount of information needed to support the algorithms that will evaluate movement over the network.

Several of the attributes are necessary to model the current state of an arc or node, vice its original state, as changes occur to the network due to obstacles or interdiction. All attributes used to describe the original state of the execution network must be explicitly input as elements of the research model data base. The attribute lists presented throughout the thesis are not 'final' in that they may be modified as development of each network and functional area continues.

1. Arcs: Uses and Attributes

a. Uses: Arcs as Routes

An arc will represent a route which is specifically identified to carry some or all products flowing in the network. The primary purpose of arcs will be to model the movement of units and logistic items in support of a given mission or tactical plan.

All arcs representative of the original transportation system will be declared prior to the execution of the model. This requires the modeler responsible for data preparation to do quite a detailed map and terrain study. Arcs will not be dynamically created during the execution of the model except for obstacle arcs which will coincide with an existing arc.

Arcs will be used to model a variety of routes, to include:

- The existing road system
- Off road, cross-country arcs
- The POL pipeline system
- The railroad system

The existing road system, (referred to as an improved road or improved arc) will be easy to designate from a detailed map study; the cross-country arcs, however, will not be as easy to identify. Because arcs will not be generated during the execution of the model, off road movement will be limited to those cross-country arcs explicitly defined. Off road arcs can only be designated after an analysis of avenues of approach has been conducted for each echelon of command.

There are several reasons why a unit would want to travel off road to reach its destination:

- The improved road has been interdicted and the obstacle must be bypassed
- It is tactically advantageous to the unit to travel off road
- The road system cannot support the unit due to the unit's travelling formation
- No road system exists to the unit's desired destination

b. Attributes:

Arc attributes must be able to support variable resolution representations of the execution network. The array of arc attributes will be described in two sections, the first reflecting the original state of the arc, the second reflecting changes made to the arc during the execution of the model. Attributes are followed by their notation and units, as appropriate.

The attributes necessary to model the initial state of an arc are as follows:

- Nodes; (x,y)
- Length; L(x,y), kilometers
- Width; W(x,y), kilometers
- Surface; SURF(x,y), discrete categories
- Mobility class; MC(x,y), discrete categories
- Off road mobility pointer; ORMP(x,y), attribute block pointer

The nodes of an arc, x and y, are used merely to designate connectivity; one end point, x, becomes an origin and the other, y, a destination when the arc is directed.

The length and width of an arc, L(x,y) and W(x,y), respectively, measured in kilometers, are the actual length and width of the segment represented by that arc. For improved arcs, width may be converted to an (integer) number of lanes, NLANES. Arcs will be required to have two or more lanes to support two-way traffic. The width of one lane, WLANE, must be specified in the data base in advance.

$$NLANES(x,y) = W(x,y)/WLANE \quad (\text{eqn 3.1})$$

The surface attribute of an arc reflects the capability of that arc to support movement by a certain type unit or product. Both improved and off road arcs have particular surface types, rank ordered, and presented in Table I. The improved arcs can support movement by all types of vehicles. The off road arcs show the maximum weight class of vehicles that can use that particular route.

The surface attribute can be used to limit the types of product flow on an arc or restrict units to arcs of a specific surface type. For example, certain arcs have a surface attribute of 'railroad' or 'pipeline', both major means of logistic resupply, but limited in the types of

TABLE I
Support Capacity of Arc Surfaces

IMPROVED ARCS
Paved, Concrete
Paved, Asphalt
Dirt Road, Gravel Based
Dirt Road

OFF ROAD ARCS: (Maximum Weight Class)
Heavy Tanks
Medium Tanks, Fighting Vehicles
Medium Trucks
Light Trucks
Dismounted Movement Only

SPECIAL CASES
Pipelines
Railroads

products that can move on them. Medium truck companies delivering ammunition to a division ASP will usually be restricted to moving on improved arcs.

The mobility class of an arc, MC, is used to describe the nature of an arc as it affects unit mobility. Mobility class is determined in part from the arc surface, but also considers features such as the slope of the terrain and relative straightness (or lack thereof) of the arc.

The off road mobility pointer, ORMP, indicates whether or not an explicit arc (x,y) has a trafficable route alongside it which is accessible from the arc. If such a route exists, its attributes are described by a new arc attribute block which is linked to the explicit arc (x,y). The off road route would not be represented as a separate explicit arc in the network, but is rather viewed as a modification of the existing explicit arc (x,y). It is understood that movement back and forth from the arc (x,y) to the off road route may occur at any point along the arc, so that

a unit may move off the road for as long as is required to bypass an obstacle, and then return to the explicit road arc. The beginning and end points of the ORMP are the same as the original arc, however, the other attributes may all change. An example of this would be an open field that is alongside a road, and can be used by selected units for movement, (at a price) off the modelled arc. The arc length need not be the same, as the ORMP may represent a more circuitous route to the same destination. The explicitly modelled arc will be the arc deemed most advantageous and the off road route(s) will be in decreasing order of preference. A secondary dirt road may be preferred less than a cross country route which offers good tactical advantages in the vicinity of the FLOT. The ORMP may recursively define multiple off road routes.

The following arc attributes are necessary to model the state of an arc as it currently affects the network.

- Maximum arc speed; $S(x,y,k)$, kilometers/hour
- Obstacle flag; $OBFLAG(x,y)$, binary (0/1)
- Attrition flag; $AFLAG(x,y)$, binary (0/1)
- Halted unit flag; $HUFLAG(x,y)$, binary (0/1)

The maximum speed for an arc, $S(x,y,k)$, measured in kilometers per hours, is a characteristic of an arc determined as a function of the surface, mobility class and the composition of a unit travelling on that arc, as shown in equation 3.2.

$$S(x,y,k) = f(\text{SURF}, \text{MC}, \text{Unit composition}) \quad (\text{eqn 3.2})$$

It is anticipated that $S(x,y,k)$ can be determined from a table look up after more work has been done to specify mobility classes.

The obstacle flag, OBFLAG(x,y) denotes arcs which are representative of an obstacle or have obstacle effects represented in its attributes. The execution network identifies the existence of such an arc; the engineer module determines the effects of the obstacle as regards necessary breaching resources, time penalties, and breaching speeds.

The attrition flag, AFLAG(x,y), will be used to indicate arcs representative of obstacles which by their nature, attrite units attempting to travel on them. This will be required to support algorithms in which attrition may be an objective function or constraint. The execution network identifies the existence of such an arc; the engineer module will determine its lethality.

The halted unit flag, HUFLAG(x,y), is similar to the obstacle flag in that it denotes arcs on which units have halted movement. The attributes of the halted unit in conjunction with the arc attributes will be used to determine any time delays and speed reductions incurred by a second, moving unit upon encountering the halted unit and will be discussed further in Section D.

Table II is a summary of the arc attributes presented above. The column heading 'units' refers to units of measurement for appropriate attributes.

TABLE II
Summary of Arc Attributes

ATTRIBUTE	NOTATION	UNITS	EXAMPLE
End Nodes	x, y	-	(x, y)
Length	$L(x, y)$	Kilometers	1.2
Width	$W(x, y)$	Kilometers	0.03
Surface	$SURF(x, y)$	Discrete	Paved, Asphalt
Mobility Class	$MC(x, y)$	Discrete	-
Off Rd. Mob. Ptr.	$ORMP(x, y)$	Pointer	1
Maximum Speed	$S(x, y, k)$	Kilometers/hour	35.00
Obstacle Flag	$OBFLAG(x, y)$	0/1	0
Attrition Flag	$AFLAG(x, y)$	0/1	0
Halted Unit Flag	$HUFLAG(x, y)$	0/1	1

2. Nodes: Uses and Attributes

a. Uses: Nodes as Locations

In the execution network, a node will reflect a fixed location that can be pinpointed in an X-Y coordinate space. A node's attributes describe why that location is of interest to the model. In effect, nodes are junction points that mark some change in the network topology. Among their uses, nodes will mark the location of:

- Permanent terrain features
- Obstacles
- Changes in the manner (mode) of product flow

Nodes will be used to locate a variety of permanent terrain features. From a map reconnaissance, the most obvious of these are road junctions and hilltops. Additionally, nodes mark any location of interest to the model where some feature of the terrain changes. A single lane road that crosses an open field and enters a wooded area would be one such example. A node is sited at the intersection of the road and the woodline to indicate a change in the nature of the terrain. Permanent man-made features, such as airfields, railroad depots, bridges, and villages are also marked as nodes.

Villages represent a special problem in connection with the level of resolution being utilized. In a corps planning module, a village might appear as a single node, important only because of its major transportation arteries. That same village in the vicinity of the FLOT represents a major obstacle to an armor battalion and might be modelled by additional nodes and arcs.

A major purpose of nodes in the execution network is to represent obstacle locations and their effects on movement as obstacles are created and reduced. This will be accomplished by changing the nature of an existing node

if that is where the obstacle occurs, or by splitting an existing arc with a new node.

Nodes will also represent locations where product flow in the network changes modes of transportation, i.e. a railroad station where goods transfer from rail cars to trucks for road movement.

b. Attributes of Nodes

The following attribute array, discussed in two sections, is necessary to describe a node.

Attributes which describe the original node must be entered explicitly for each node in the original network and are described below.

- Location; LOC(z), geographic coordinates
- Classification; CLASS(z), discrete categories
- Length; L(z), measured in kilometers
- Vehicle capacity; VC(z), measured as number of vehicles
- Alternate route pointer; ALTRTE(z), attribute block pointer

The location of a node will be measured in an X-Y coordinate space and reflects actual geographic location.

Classification of a node reflects 'what' the node represents; Table III is a partial list of node classifications. Classification will be used to differentiate between permanent and temporary nodes in the network. A permanent node will represent those terrain features which can be altered but not removed from the network. A temporary node will be inserted or removed from the network as obstacles are created or reduced.

The length of a node, L(z), measured in kilometers, is the size of the feature represented by the node. Node length will be insignificant for 'small' nodes where its length can be added to the incident arc lengths, however

TABLE III
Node Classifications

PERMANENT TERRAIN FEATURES

Road Junctions	Bridges
Autobahn Interchanges	Hilltops
Villages	

OBSTACLES

Tank Ditches	Contaminated Areas
Log Barriers	Minefields
Road Craters	Nuclear Blowdown
Bridges (destroyed)	

CHANGE OF TRANSPORTATION MODE

node length will be required when the node represents a larger feature. It is recognized that nodes traditionally represent points and require no time to cross. In the research model, however, node length is a concept necessary to support node representation of such features as villages in low resolution models. If it becomes necessary, the node can be expanded to represent its detail and explicitly model movement through it.

Vehicle capacity, VC, represents the maximum number of vehicles, moving or stopped, which may be considered 'inside' a node at any one time. VC will be required when modelling the movement of multiple units entering a node. Procedures to determine VC have not yet been determined.

The alternate route pointer, ALTRTE, indicates whether or not a secondary route is available for movement through a node after the main route has been interdicted. For example, it may be possible to move through a village (represented by a single node) on alternate routes after the main avenue has been cratered. Use of ALTRTE will allow an

exit on any arc incident to that node. The recursive declaration of alternate routes is analogous to the declaration of off road mobility routes.

There are additional node attributes which are necessary to represent the current state of a node. These attributes are:

- Maximum node speed; $S(z,k)$, measured in kilometers per hour
- Current vehicle count; $CVC(z)$, measured in number of vehicles
- Obstacle flag; $OBFLAG(z)$, binary (0/1)
- Obstacle length; $Lo(z)$, measured in kilometers
- Obstacle speed; $So(z)$, measured in kilometers per hour
- Attrition flag; $AFLAG(z)$, binary (0/1)
- Halted unit flag; $HUFLAG(z)$, binary (0/1)

The maximum speed through a node, $S(z,k)$, in kilometers per hour is the maximum velocity that node can support. In most cases, node velocity will be equal to the maximum arc velocity of at least one incident arc. In all other cases, such as a node representative of a village, maximum node velocity is a function of node classification and the unit type crossing that node, shown in equation 3.3, which may be determined by a table look up.

$$S(z,k) = f_1(\text{CLASS}, \text{Unit Composition}) \quad (\text{eqn 3.3})$$

The current vehicle count, $CVC(z)$, represents the number of vehicles currently moving or stopped inside a node. CVC will be used in discrete movement schemes and is necessary to insure that a node's capacity is not exceeded. Movement algorithms will establish a priority of node use for various units; depending on the algorithm derived, multiple units may be allowed to pass through a node simultaneously or as one unit following another. One technique

for measuring CVC is based on a unit's density, UD, measured in vehicles per kilometer, as shown in equation 3.4. Equation 3.4 assumes that the length of unit (k) in node (z) is as long as node (z), which prevents a unit from entering node (z) if CVC will exceed VC.

$$CVC(z) = \sum_k UD(k) \times L(z) \quad (\text{eqn 3.4})$$

The obstacle flag, OBFLAG(z), is used to indicate the presence of an obstacle at node (z). The node may be a permanent node at which an obstacle was placed, or a temporary node representing only an obstacle and no other terrain feature. The effects of the obstacle at node (z) are described by the engineer module. Obstacle length, Lo(z), measured in kilometers, is the length of the obstacle as seen by the approaching unit. The obstacle speed, So(z), represents the maximum allowable speed while crossing an obstacle, measured in kilometers per hour, and when the obstacle is placed at a permanent node, takes priority over the node's maximum (unaltered) speed.

The attrition flag, AFLAG(z), applies to those obstacles which, by their nature, cause attrition to a unit attempting to breach them and is identical to the arc attrition flag.

The halted unit flag, HUFLAG(z), is defined similar to the halted unit attribute of an arc and identifies the presence of a unit stopped at node (z). HUFLAG will be necessary when determining if time penalties due to congestion are assessed at a node. Determination of the time penalties will be discussed in Section D.

Table IV is a summary of the node attributes which have been discussed above. The column heading 'units' refers to units of measurement for appropriate attributes.

TABLE IV
Summary of Node Attributes

ATTRIBUTE	NOTATION	UNITS	EXAMPLE
Location	LOC	Geo. Coordinates	1368
Classification	CLASS	Discrete categories	Rd Jcn
Length	L(z)	Kilometers	0.01
Vehicle Capacity	VC(z)	No. of vehicles	5
Alternate Route Pointer	ALTRTE(z)	Pointer	1
Maximum Node Speed	S(z,k)	Kilometers/hour	35.0
Current Vehicle Count	CVC(z)	No. of vehicles	0
Obstacle Flag	OBFLAG(z)	0/1	1
Obstacle Length	Lo(z)	Kilometers	0.01
Obstacle Speed	So(z)	Kilometers/hour	6.00
Attrition Flag	AFLAG(z)	0/1	0
Halted Unit Flag	HUFLAG(z)	0/1	0

D. PLANNING NETWORKS

The purpose of a planning network is to provide a structure on which continuous flow optimization algorithms can be applied in projecting future combat operations. The planning networks are intended to be abstractions of the more detailed execution network. There will be several planning networks in use at any one time, each addressing the needs of a specific unit in the hierarchical force structure and representing that unit's current sector of interest.

1. Extraction of Planning Networks from the Execution Network

It is currently envisioned that avenues of approach, which are terrain routes that can support the movement of a given unit, will be input as part of the initial data base and have their boundaries defined by a series of X-Y coordinates. Avenues of approach will be designated following a detailed terrain analysis without regard to the underlying network structure. An avenue of approach which supports a larger unit such as a division, will be further subdivided to reflect avenues that support the division's subordinate regiments.

Through a process known as 'templating', units will be allocated to defend a possible avenue of approach, again without regard to the underlying network. Unit sectors, which will vary with mission and current unit location will be as wide as the avenue of approach on which the unit is placed and include front and rear unit boundaries.

The portion of the execution network which will be extracted to form the planning network underlies the current unit sector. The planning network will be constructed by extracting specially coded arcs and nodes from the execution network. The necessary codes will be carried in the

attribute arrays of the arcs and nodes that have already been presented.

For example, a corps planning network interested in estimating the maximum arrival rate of combat units at the FLOT might extract only the paved, improved roads from the execution network. A battalion planning network seeking the shortest path for movement to a new location might extract all arcs classified as 'Medium Tanks, Fighting Vehicles' or better from the execution network.

2. Attribute Coding Requirements

Since the planning networks are to be extracted from the execution network, the arc and node attributes in the execution network must be stored in a manner which allows them to be used in optimization routines found in the planning algorithms. This section discusses special coding requirements of the attributes of the execution network.

a. Arc Attributes

Arcs will serve the same purpose in planning networks that they serve in the execution network. Depending on the unit doing the planning and its purpose for planning, however, the number of arcs and the size of the sector being modelled should be significantly reduced in the planning network. Those arc attributes of the planning network which are defined differently from the execution network, or demand special coding are discussed below.

The surface attribute of an arc must be coded in such a manner that it indicates the highest unit in the force structure which would utilize that arc in an optimization routine and is understood to apply to all other units below that unit in the hierarchy. For example, a multilane highway would appear in the planning networks of all units from battalion to corps; a narrow road might only appear in a battalion planning network.

When the surface attributes are coded, the modeler must insure that the connectivity of the area being represented is maintained in the planning network. This might cause an arc such as a narrow dirt road connecting two paved roads to be represented in the corps planning network.

b. Node Attributes

Nodes in a planning network will represent the same features they represent in the execution network. However, nodes in a planning network must be points without length and time attributes, so that continuous flow optimization algorithms can be used. The following relation will be used to transfer node length to the arcs for all arcs (x,y) when node length $L(x)$ or $L(y)$ is greater than zero. As shown in equation 3.5, each arc is assigned one half of the length of each end node. $La(x,y)$ represents the length of arc (x,y) and used in lieu of $L(x,y)$.

$$La(x,y) = L(x,y) + .5(L(x) + L(y)) \quad (\text{eqn 3.5})$$

The speed associated with a node $S(x,k)$ or $S(y,k)$ will be represented on arc (x,y) in the planning network as the weighted average of the speeds on the arc and through the nodes. The sum of the times required by unit k to travel arc (x,y) and one half the time to travel node (x) or (y), represented by $Tl(x,y,k)$, is computed in equation 3.6. Equation 3.7 computes the average speed on an arc (x,y), which is represented by $Sa(x,y,k)$. $Sa(x,y,k)$ is used in the planning network in lieu of $S(x,y,k)$ and the node speeds, $S(x,k)$ and $S(y,k)$, are disregarded.

$$Tl(x,y,k) = L(x,y)/S(x,y,k) + .5(L(x)/S(x,k) + L(y)/S(y,k)) \quad (\text{eqn 3.6})$$

$$Sa(x,y,k) = La(x,y)/Tl(x,y,k) \quad (\text{eqn 3.7})$$

This method of representing arc speed in the planning network is identical to explicitly representing an arc of one-half the node length at each end of an arc (x,y) without changing the connectivity of the network.

E. UNIT REPRESENTATION ON THE TRANSPORTATION NETWORK

The primary purpose of the execution and planning networks are to provide the mechanism by which units move around the battlefield. Two techniques for modelling unit movement and location on the transportation network will be discussed in this section. The first method represents a unit as a point mass and is used in conjunction with the planning network. The second method represents a unit as a line segment by recording the position of the unit's head and tail elements and is used on the execution network.

1. Necessary Unit Information

The units that travel on the execution network must have certain attributes, listed below, for use in the movement algorithms.

- Unit type; $UT(k)$, discrete categories
- Intervehicle spacing; $Ls(k)$, kilometers
- Number of vehicles of type j ; $NVEH(j,k)$,
- Movement priority; $MP(k)$
- Dispersion (formations); $DISP(k)$, discrete categories
- Allowable arc surfaces
- Passage speed; $Shu(k)$, kilometers per hour
- Passage delay; $Thu(k)$, hours
- Unit head location; $ULOCH(k)$, kilometers on arc (x,y)
- Unit tail location; $UOCT(k)$, kilometers on arc (x,y)

Unit type, UT, describes what makes a specific unit interesting to model. For example, unit type distinguishes between armor battalions and detachments which operate corps ammunition supply points. Unit type also identifies a key vehicle type for use in determining maximum arc and node speeds.

The intervehicle spacing, L_s , measured in kilometers, is the desired spacing between vehicles. L_s may be changed by a unit's planning module as a result of a change to the unit's mission. The minimum length of a unit, $L_u(k)$, measured in kilometers, represents the desired 'first to last vehicle' length, as shown in equation 3.8. $L_v(j)$ is defined as the length of each vehicle type j and $NVEH(j,k)$ is the number of vehicles of type j in a unit (k) .

$$L_u(k) = \sum_j NVEH(j,k) \times (L_v(j) + L_s(k)) \quad (\text{eqn 3.8})$$

The current unit length, $L_{uc}(k)$, measured in kilometers, will be allowed to vary over time and will be discussed later in this chapter.

A unit receives a movement priority, MP, on the road network based on the current situation and future operation. Priorities may be specified by the appropriate planning module for a time block or the duration of an operation.

The dispersion attribute, DISP, indicates which travelling formations a unit is allowed to assume while travelling cross-country. Unit formations and arc widths will be necessary to evaluate off road avenues of approach for combat units. A unit is assumed to be in a trail formation, i.e. one vehicle behind another, while moving on improved arcs.

The allowable surface attribute restricts vehicles and units to routes of specified surface type.

Passage speed, $Shu(k)$, measured in kilometers per hour, and passage delay, $Thu(k)$, measured in hours, represent the effects of unit (k) to the network given the unit has halted its movement. $Shu(k)$ then is the speed a unit $(k1)$ may pass through unit (k) ; $Thu(k)$ is a discrete time penalty incurred by unit $(k1)$ as a result of meeting unit (k) on an arc or at a node. Shu and Thu are functional relationships dependent on the halted unit's composition and the nature of the arc(s) or nodes(s) at which the unit has halted. Equations 3.9 and 3.10 show the relation for Shu for a unit halted on an arc and at a node, respectively.

$$Shu(x,y,k)=f3(UT,NLANES,SURF,MC,ORMP) \quad (eqn\ 3.9)$$

$$Shu(z,k)=f4(UT,CLASS,ALTRTE) \quad (eqn\ 3.10)$$

Equation 3.11 shows the relation for Thu and is applicable to both arcs and nodes. It is anticipated that Shu and Thu will also be determined via a table look-up. Rules for multiple units on an arc will be addressed later in this section.

$$Thu(k)=f5(UT,Operational\ Mission) \quad (eqn\ 3.11)$$

Unit location in the execution network may be monitored by recording the location of both the head and tail vehicles of a unit, $ULOCH(k)$ and $ULOCT(k)$, respectively, along with a sequence of arcs designated as the unit's path. Unit location will be measured in kilometers in terms of the arc(s) on which the head and tail are located.

$ULOCH$ and $ULOCT$ will be measured from the originating node of the arc(s) on which the first and last vehicles are located. A unit located entirely on one directed arc (x,y) will have both its head and tail vehicles measured

in terms of distance from node x . If a unit is spread over two or more adjacent arcs (x,y) and (y,z) and directed from node x to node z, then the unit head will be measured from node y and the unit tail measured from node x. A head or tail vehicle at a node is considered at the destination node of an arc. Unit dispersion between the head and tail is assumed uniform for the entire length of the unit.

2. Representing a Unit on the Planning Network

Representing a unit as a point mass has several advantages in terms of simplicity and information storage requirements. Point representations are useful for projecting a unit's future location: friendly unit movement for planning future operations and enemy movements for planning interdiction operations to the network. The point mass method will also be used to track enemy units beyond the range of friendly weapons systems but within range of intelligence gathering assets. A point mass will be required when solving shortest path problems.

When a unit is modelled as a point representation, the movement of that unit is restricted only by the maximum arc speed $S(x,y,k)$. The beginning unit location in a planning network will correspond to the location of the unit's head on the execution network. The minimum time for unit k to travel the length of an arc, $T(x,y,k)$, measured in hours, is shown in equation 3.12.

$$T(x,y,k) = L(x,y)/S(x,y,k) \quad (\text{eqn 3.12})$$

Assuming that there are no time penalties due to obstacles movement time on a given path for a unit (k1), $TP(A,k)$, measured in hours, can be computed as shown in equation 3.13.

$$TP(A,k) = \sum_{(x,y)} T(x,y) \quad \forall (x,y) \in (A) \quad (\text{eqn 3.13})$$

3. Representing A Unit on the Execution Network

A line segment representation of a unit has advantages in depicting a unit's current location on the execution network. The end points of the segment will correspond to ULOCH and ULOCT. Properties of the unit head, (represented by the notation (kh)), moving on the network are the same as those developed for a unit moving as a point mass.

$Lu(k)$ was introduced as a unit's minimum length; that is, the length a unit should maintain while travelling. However, current unit length, $Luc(k)$, will vary (with $Lu(k)$ as a minimum) as the unit head and tail vehicles travel on different arcs and nodes.

If the current unit length equals the minimum unit length, then the unit will start and stop instantaneously. If $Luc(k)$ is greater than $Lu(k)$ when the head halts movement, the tail will continue to travel until the unit's length is again equal to $Lu(k)$.

The disadvantage of modelling a unit as a segment lies in the increased computations necessary to model the movement of the unit tail. Unit length must be computed at the end of each (time or event) step before a tail speed can be determined for the next step. Equation 3.14 is used to determine current unit length if unit (k) is located entirely on one arc; equation 3.15 is used otherwise. The index of all nodes on the unit's path between ULOCH and ULOCT, excluding any nodes at which the head or tail may be located is designated by i . The index of the originating node of the arc on which the unit head is located is noted by I .

$$Luc(k) = ULOCH(k) - ULOCT(k) \quad (\text{eqn 3.14})$$

$$Luc(k) = HEAD + \sum_{i=1}^{x-1} L(x_i, x_{i+1}) + \sum_{i=1}^x L(x_i) + TAIL \quad (\text{eqn 3.15})$$

HEAD and TAIL represent the partial length of the arcs or node at which the head and tail are currently located. If the unit head is on an arc, HEAD is calculated as shown in equation 3.16. If the unit head is at a node, and the time spent by the unit crossing that node, TNODE(z), is less than T(z), the total time to cross a node, HEAD is calculated as shown in equation 3.18.

$$HEAD = ULOCH(k) \text{ if } k \text{ is on an arc} \quad (\text{eqn 3.16})$$

$$T(z, k) = L(z) / S(z, k) \quad (\text{eqn 3.17})$$

$$\text{If } k \text{ is at a node, and } TNODE(z) < T(z) \quad (\text{eqn 3.18})$$

$$HEAD = TNODE(z) \times S(z, k)$$

Equations 3.19 and 3.20 are similarly described for TAIL, the distance remaining before unit k is off of arc (y, z) or clear of node (z).

$$TAIL = L(y, z) - ULOCT(k) \text{ if } k \text{ is on an arc} \quad (\text{eqn 3.19})$$

$$\text{If } k \text{ is at a node and } TNODE(z) < T(z) \quad (\text{eqn 3.20})$$

$$TAIL = L(z) - TNODE(z) \times S(z, k)$$

Developing an algorithm which determines the speed of the unit tail will be very difficult. Several situations have been identified which must be considered. It is assumed that one goal of unit movement is to arrive at the destination in the minimum time. Therefore, the unit head will move at the arc or node maximum speed on which it is located. Selecting the current speed of the unit tail however, depends on identifying that arc or node between the head and tail vehicles which is dictating the tail speed. There may exist an arc or node between the head and tail

with a maximum speed less than the maximum speeds of the arcs or nodes on which the head and tail are located. This 'slower arc' would continue to hinder movement of the tail while the unit head could be moving much faster than the remainder of its unit. In another situation, the unit head may be stopped at an obstacle, and it must be determined if the tail is moving at maximum speed or also hindered by a second obstacle between the head and tail.

Modelling a unit's length will be important in algorithms which evaluate the effects of interdiction against that unit. Rules and algorithms will need to be developed for use in modelling a unit's length and the speed of the tail vehicle.

4. Multiple Units on the Execution Network

As units move around the execution network, there will often be a need to plan for multiple units on an arc or path and for two-way traffic along one arc. Two-way traffic is often necessary along main supply routes running the length of a unit's sector.

Units will be required to maintain unit integrity when travelling and when overtaking another unit. The situation may occur when a faster moving unit (k_1) of higher priority is behind another unit, (k). The trailing unit will be allowed to pass only when the entire leading unit has halted its movement. The movement algorithm will identify the conflict in routing and issue the order to unit (k) to stop its movement. The time penalties, if any, are calculated for the length of the arc(s) and node(s) over which the passage takes place. Discrete penalties are represented by $Thu(k)$, and the reduced speed of unit (k_1) is equal to $Shu(k)$. The time required for unit (k_1) to traverse that portion of a path affected by a halted unit (k), $TUSTOP$, is shown in equation 3.21. I and i are defined as before.

$$\begin{aligned} \text{TUSTOP} = & \text{Thu}(k) + \sum_{i=1}^{I'} L(x_i, x_{i'}) / \text{Shu}(x_i, x_{i'}, k) \\ & + \sum_{i=1}^I L(x_i) / \text{Shu}(x_i, k) \end{aligned} \quad (\text{eqn } 3.21)$$

Two-way traffic is a modelling problem in movement routines. In a steady state optimization model of a single commodity network, unit flow in opposite direction has the net effect of reducing the overall flow rate. Because this presents an incorrect representation, continuous state flow models of two-way traffic in the network have limited applicability.

F. OBSTACLE REPRESENTATION ON THE TRANSPORTATION NETWORK

Obstacles which change the current state of the network will be represented by one of three modelling methods. In the execution network, an obstacle can be represented as an additional node or arc in the network or as a change to the attributes of the existing nodes and arcs. In the planning network, obstacles will only be represented as changes to the attributes of the existing arcs and nodes.

The selection of one obstacle representation over another in the execution network should depend in part on the level of resolution being used and the goal of the study. If one wishes to study the effects of obstacles on maximum flow or minimum cost, then obstacles can be represented in the attributes of the original network. If one wishes to study the decision algorithms of actions at an obstacle and the effects of these decisions, then the obstacles need to be represented as explicitly as possible. In most cases, obstacles will be represented as nodes in the execution network.

1. Needed Obstacle Information

The engineering module being developed to support the research model will define the resources required to emplace or reduce a certain type obstacle, as well as the effect that obstacle should have on movement in the network. The movement algorithms must be supplied information from the engineering module concerning:

- Obstacle location
- Obstacle type; OT
- Obstacle length; L_o , kilometers
- Obstacle speed, S_o , kilometers/hour
- Attrition flag; AFLAG, binary (0/1)
- Unit delay time; T_d , hours
- Breach delay time; T_b , hours

The obstacle effects to the network have been previously defined as attributes for arcs and nodes. When an obstacle is modelled as an arc, obstacle length L_o and obstacle speed S_o are stored as the arc length attribute $L(x,y)$ and maximum arc speed $S(x,y,k)$ respectively. When an obstacle is modelled as a node, L_o and S_o are stored as obstacle length $L_o(y)$ and obstacle speed $S_o(y,k)$, respectively.

The unit delay time, T_d , and the obstacle breach time, T_b , represent the discrete delays in hours caused by an obstacle and are assessed against a unit moving through the obstacle. The unit delay time, T_d , represents the time delay incurred when a unit first arrives at an obstacle. This would include the time required to evaluate the obstacle and decide whether or not to breach it. The attribute will carry a flag which indicates if the delay applies to only the first unit to arrive at the obstacle or to all units, and for a first-unit-only delay, T_d will be changed to zero after it has been assessed. The breach delay time, T_b , is the time required to actually breach the obstacle and

create a single lane for traffic. For example, this would model the time required to emplace an AVLB (Armor Vehicle Launched Bridge) across a road crater or to clear a lane through a minefield. T_d and T_b are functional relationships dependent on the interaction of several factors, as shown in equation 3.22 and 3.23, and may become data available via a table lookup.

$$T_d = f_6(UT, OT) \quad (\text{eqn 3.22})$$

$$T_b = f_7(UT, OT, \text{SURF}, MC, Lo, AFLAG) \quad (\text{eqn 3.23})$$

2. Obstacle Modelling Methods

There are several reasons why one would interdict an arc; the end result is directed toward reducing the enemy's unit value. For example, interdiction of the railroad network reduces a force's ability to sustain itself. Interdiction of the road and cross-country routes can increase a unit's travel time or aid in its destruction. Some obstacles are placed to completely deny a route; these obstacles should be on an arc with no off road mobility. Other obstacles are placed to restrict the attacking force to movement over terrain selected by the defender.

This section will discuss representing obstacles at either nodes or on arcs using each of the three methods. The discussion will address both inserting and removing the effects of obstacles. Reducing an obstacle reverses the constraining process until the arc or node is back to its pre-obstacle state.

a. Obstacle Representation on an Arc

The most explicit method of modelling an obstacle, and also the most difficult, is to represent the

obstacle as a new arc located along an existing arc. This requires the addition of two nodes and one arc to the execution network for each obstacle created. Figure 3.1 shows the process of inserting an obstacle represented by arc (b,c) into an existing arc (x,y). At step 1, node b of arc (b,c) is inserted into arc (x,y), creating two new arcs, (x,b) and (b,y), each with attributes similar to the original arc (except length). This process is repeated at step 2, where arc (b,y) is now split into arcs (b,c) and (c,y). Arc (c,y) has attributes similar to the original arc; arc (b,c) has its attributes, representative of the obstacle, supplied by the engineer module.

In Figure 3.1, the adjacent node to each end node is written as a subscript at the end of each arc to show the several changes that have to be made to the connectivity scheme as the network changes.

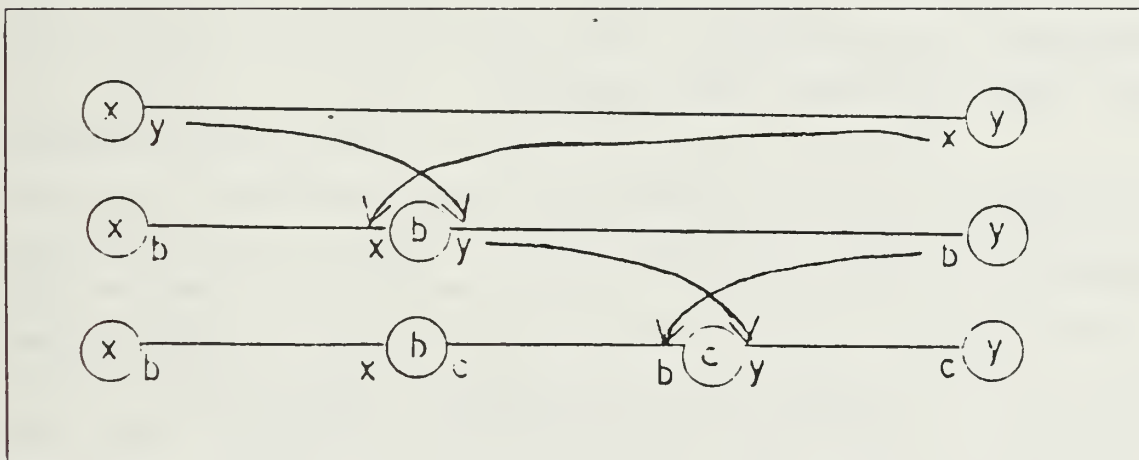


Figure 3.1 Arc Representation of an Obstacle

The new obstacle arc continues to lengthen as the obstacle is improved by the application of additional engineering resources. Assuming that resources are committed as discrete packets of interdiction, the location

of the end node(s) of the obstacle arc changes after every step causing changes to the attributes of at least one node and two arcs. This process reverses itself as the obstacle is reduced by mobility operations. Other obstacles are improved not by lengthening them, but by enhancing the means by which their maximum arc speed is reduced or attrition is increased. Reduction of this type of obstacle causes it to 'fade away', i.e. lose its value, rather than shorten with each step.

Representing an obstacle as an arc appears to have value for modelling extremely large obstacles such as minefields, chemically contaminated areas or areas rubbleed by nuclear weapons. These types of obstacles are usually breached by one or two lanes which are narrow relative to the size of the obstacle (minefield) or are crossed after special preparations by the unit which do not change the nature of the obstacle (contaminated areas). Unit location can be modelled on the obstacle arc as the unit crosses the obstacle. Additionally, that unit can be taken under fire at a known location in the obstacle.

Using a node to represent an obstacle is a more efficient means of representation than an arc, and portrays the same movement effects. Using a node to represent an obstacle on an arc requires that a new node be placed on the original arc at the location of the obstacle. The arc is split as shown in step 1 of Figure 3.1 and results in one additional arc and node in the execution network. The node is passed its attributes from the engineer module; the two new arcs assume the attributes of the arc which was split.

If an existing obstacle represented by a node is improved or reduced, the changes can be recorded in the node attributes with no additional changes to the structure of the network.

Representing an obstacle on an arc as a node is an effective technique for modelling point obstacles designed to restrict an arc rather than deny a large area. Point obstacles are constructed and reduced during the course of a battle, are rarely permanent, and are the majority of the obstacles found on the battlefield.

This method, however, does not explicitly model a unit's location as it crosses large obstacles, and the unit cannot be interdicted at specific points within an area type obstacle.

The third method of representing an obstacle on an arc is to reflect the obstacle's effects within the attributes of the existing arc without altering the network structure. The obstacle can be reflected either as a change to the maximum arc speed $S(x,y,k)$ or as a discrete unit delay time $T_d(x,y)$, added to the unit's passage time along the arc.

Representing an obstacle in the attributes of an arc will be utilized as the method of representing obstacles in planning networks concerned with shortest path and maximum flow.

Because the exact location of the obstacle on the arc is not recorded, this technique is not suitable for the execution network which must represent unit actions and decisions at an obstacle.

b. Obstacle Representation at a Node

There are several reasons why one would want to interdict a node. Successful interdiction efforts at a node could deny the enemy force use of several arcs incident to that node at once, thereby affecting shortest path times and unit values. Interdicting nodes could also deny key terrain to the enemy (reducing the effects of his weapons) and limit his ability to use key logistical centers (airfields, train

stations). Only two modelling methods, changing the attributes of the existing node and representing the obstacle as an arc are appropriate for use at a node.

Representing an obstacle at a node by changing the attributes of that node is both fast and simple using the data provided by the engineer module. The original node is already described in both the execution and planning networks. The only changes occur within the attribute array of one node and not to the structure of the network.

Representing an obstacle at a node via the node's attributes is best suited for small, point obstacles that are within the bounds of the terrain feature represented by the node. This method can also be used to represent a large area obstacle that interdicts a node representative of an equally large feature (e.g. a chemical strike on an airfield).

The second method of modelling an obstacle at a node uses an arc to represent the obstacle. This method is very inefficient and will not be used. It is presented here merely as an alternate modelling method. Changing the network structure involves inserting at least two additional nodes into the network, one either side of the original node, and splitting the original arcs entering and leaving the node. For nodes with more than two incident arcs the orientation of the obstacle must also be specified.

Figure 3.2 shows the effect to a simple network of representing an obstacle at a node by arcs. The nodes m, n, y, and z were all added to the network as outlined for step 1, Figure 3.1. The number of arcs in this current network has doubled and every existing node and arc has had at least one attribute changed. Additionally, the original nature of node x must be preserved (possibly as an additional block of attributes associated with node x) so that the four original arcs and node x can be restored if the obstacle is reduced.

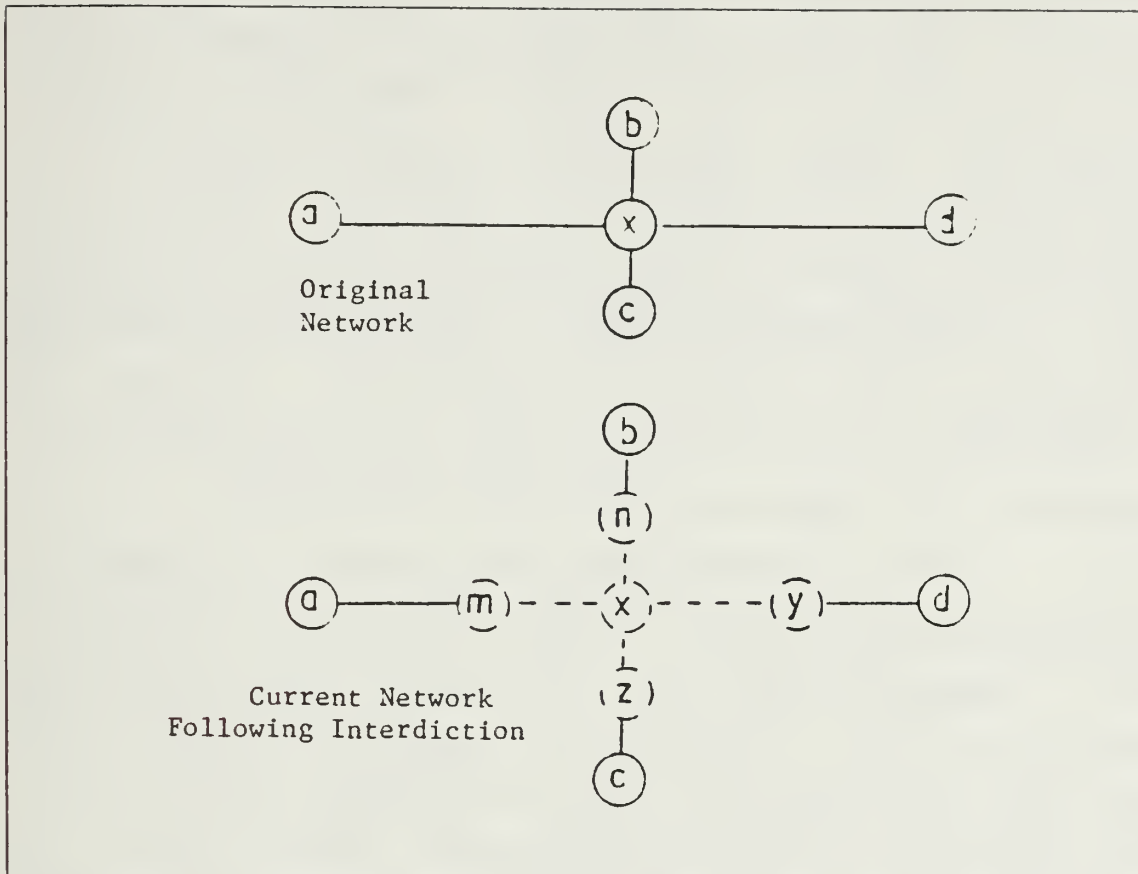


Figure 3.2 Arc Representation of an Obstacle at a Node

There are a few occasions where this method could be used efficiently. For example, clearing rubble which was formed by destroying a highway interchange opens one arc while the elevated arc remains closed. Also, this method could be used in lieu of adding attributes to nodes solely to model obstacle effects. The additional attributes would have to be carried by each node when only a small proportion of nodes will eventually be interdicted.

3. Measuring Obstacle Effects

It will be computationally inefficient to measure the effects of obstacles in either the execution or planning

network by recomputing the transit time of a given path, or to resolve for the shortest path following insertion of an obstacle. The procedure recommended here is to calculate only the time delay, TDEL, (hours), caused by the obstacle to path (A). Define path (A') to be that portion of path (A) affected by the obstacle. In most cases, (A') will be a single arc or node; if the obstacle is placed on an arc, then A' is determined as follows. The originating node for path (A') is the permanent node immediately preceding the (temporary) obstacle node (or nodes if the obstacle is modelled as an arc). The end node for path (A') is the permanent node immediately following the obstacle node or nodes. Let path (A'') model that same portion of the network as path (A') with the connectivity of path (A'') representing insertion of the obstacle. TDEL, calculated in equation 3.24, represents the time value of an obstacle to the emplacing unit and a similar time penalty to the unit which must cross that obstacle. TDEL can be computed for a single arc, TDEL(x,y), or for a single node, TDEL(z). TP, the unobstructed time on a path, was calculated in equation 3.13.

$$TDEL = TP(A'', k) - TP(A', k) + \sum_{\substack{\text{arc} \\ \text{in PATH } A''}} \sum_{\text{node}} (T_d + T_b) \quad (\text{eqn 3.24})$$

Off road mobility and alternate route pointers were described as attributes for an arc and node, which would allow a unit to bypass an obstacle. Given that an arc with an ORMP has been interdicted, bypassing the obstacle is preferred to breaching operations when equation 3.25 is satisfied. Equation 3.25 assumes bypass decisions are made instantly and no time delay penalty is assessed to the unit.

$$\begin{aligned} (L(x,y) - L_o(z)) / S(x,y,k) \\ + L_o(z) / S_o(z,k) < TP(A'', k) + T_b(x,y) + T_d(x,y) \end{aligned} \quad (\text{eqn 3.25})$$

By comparing the total travel time of path (A") with the travel time using the off road route, the entire path time need not be recomputed.

Nodes in the planning network can not have attributes of length or time. In a manner similar to equation 3.5, one-half of the discrete time delay incurred at a node will be added to each arc incident to that node, as shown in equation 3.26; T_a represents the adjusted arc time.

$$T_a(x,y,k) = T(x,y,k) + .5(T(x,k) + T_b(x) + T_d(x)) \quad (\text{eqn 3.26}) \\ + .5(T(y,k) + T_b(y) + T_d(y))$$

G. MODELLING FLOW ON THE PLANNING NETWORK

This section is designed to show the applicability of using continuous flow models on the planning network.

1. Use of Homogeneous Flow Models

Continuous flow models are useful for modelling movement of aggregated forces on planning networks. Flow models can represent the arrival of combat forces at the FLOT (or some other area) or the movement of logistic items around the battlefield over a long period of time. In a low resolution model, combat forces and logistic items can be thought of as homogeneous flow measured in vehicles per hour or tons of supplies per hour, respectively. Numerous flow optimization algorithms exist which efficiently solve for the maximum flow or minimum cost flow in a network. The following formulation models the homogeneous flow of combat forces to the FLOT and is based on the algorithm presented in [Ref. 7: p. 63].

a. Assumptions

- All flow originates from, and no flow enters the source node, (s)
- All flow terminates at, and no flow leaves the sink node, (t)
- The network defined by all arcs (x,y) is a subnetwork of the larger, original network and represents the unit section being modelled.
- Vehicle length, L_v , and intervehicle spacing, L_s , remain constant
- Unit road march speed, S_u , is constant for all vehicles

b. Notation

- $F(s,t)$ Amount of flow, measured in vehicles per hour, from (s) to (t)
- $f(x,y)$ Flow on arc (x,y) measured in vehicles per hour, decision variable to be determined
- $c(x,y)$ Capacity of arc (x,y) measured in vehicles per hour, computed below in equation 3.27
- (s) Source node, input from planning module
- (t) Sink node, input from planning module
- L_v Vehicle length, input from data base
- L_s Intervehicle spacing, input from unit attribute list
- $S(x,y,k)$ Maximum arc speed, computed in equation 3.2

c. Formulation

Arc capacity, $c(x,y)$, which represents the upper bound for the flow on an arc $f(x,y)$ is calculated as shown in equation 3.27.

$$c(x,y) = S(x,y,k) / (L_v + L_s) \quad (\text{eqn 3.27})$$

In the following formulation, $f(x,y)$ represents the decision variable to be determined for each arc, measured in vehicles per hour.

$$\begin{array}{ll} \text{maximize} & F(s,t) \\ \text{subject to} & \end{array} \quad (\text{eqn 3.28})$$

$$\sum_{\substack{x \text{ such that} \\ (s,x) \text{ in network}}} f(s,x) = F(s,t) \quad (\text{eqn 3.29})$$

$$\sum_{\substack{y \text{ such that} \\ (x,y) \text{ in network}}} f(x,y) - \sum_{\substack{j \text{ such that} \\ (j,x) \text{ in network}}} f(j,x) = 0 \quad \forall \text{ node } x \text{ in network} \quad (\text{eqn 3.30})$$

$$\sum_{\substack{y \text{ such that} \\ (y,t) \text{ in network}}} f(y,t) = F(s,t) \quad (\text{eqn 3.31})$$

$$0 \leq f(x,y) \leq C(x,y) \quad (\text{eqn 3.32})$$

d. Discussion

Routines to solve the maximum flow problem are well known and computationally efficient, [Ref. 6], and [Ref. 7]. Most common algorithms use an iterative process of 'labelling' and 'scanning' nodes in a search for a flow augmenting path. Convergence is guaranteed after a finite number of iterations, [Ref. 9: p. 145], and if an optimal solution exists, the source node will be labelled and the sink node unlabelled following the last iteration. This information will be useful when 'cut sets' are discussed.

If the network has more than one actual source, say (s_1) and (s_2) , then an artificial source (s) is designated with arcs to each actual source node i.e. (s,s_1) and (s,s_2) , and each added arc has unrestricted flow. In the

final solution to the problem, $f(s,s_1)$ and $f(s,s_2)$ each represent the flow originating from the actual source nodes (s_1) and (s_2). Multiple sink nodes are modelled in a similar fashion.

The formulation has assumed that units are homogeneous and that a maximum arc speed, $S(x,y,k)$, can be calculated. Due to the complex nature of a combat unit, a process will have to be developed to account for the characteristics of the various vehicles in the unit. Additionally, it may be necessary to designate a notional vehicle of length L_v , against which all other vehicles are compared. Identifying a maximum node and arc speed will be difficult to accomplish, but necessary before arc and node capacities can be calculated.

Maximum flow algorithms assume that nodes are unrestricted in their amount of flow. It will often be the case in the research model, however, where capacitated nodes have an affect on the model. Price, [Ref. 7: p. 70], describes one method where a capacitated node (x) is divided into two nodes (x_1) and (x_2) and the effects of node (x) are modelled on arc (x_1, x_2). All arcs (j, x) entering (x) in the original network are incident to node (x_1) and all arcs (x, y) exiting (x) are incident to (x_2). This method quickly increases the size of the network, especially if arcs incident to capacitated nodes are undirected. However, this procedure will be used in the research model; the alternative requires that two additional constraints be added to the formulation. First, that the flow on all arcs (x, y) exiting from node (x) be less than the capacity of node (x) and second, that the sum of the flows on all arcs (x, y) be equal to the flow into node (x).

Obstacles inserted in the network may also have an affect on the maximum flow rate, $F(s,t)$. As previously stated, obstacles in a planning network have their effects

aggregated into the attributes of the arc on which they are placed. The effects of an obstacle which capacitates an existing node (x) has been discussed above.

e. Arc Interdiction to Disrupt Maximum Flow

In military operations, plans to interdict the network are formulated whether or not the enemy situation is well known. In the case where the enemy status is not clear, initial planning would be directed at reducing the capacity of the network flow of enemy units towards the friendly positions. If a commander has a single obstacle with which he can interdict the maximum flow by placing it on one arc, it is obvious that that obstacle would be placed on the arc where the maximum flow rate $F(s,t)$ could be reduced by the greatest amount. Those arcs on which one would consider placing that obstacle can be identified by use of the Max Flow - Min Cut Theorem, as discussed in [Ref. 9: p. 147].

Given a network with source node (s) and sink node (t), divide the set of nodes into two arbitrary, disjoint subsets, C and C', such that (s) is a member of C and (t) is a member of C'. The set of arcs from C to C' is known as a 'cut' and is denoted as (C,C'). The capacity of the cut is the sum of the capacities of each arc, $c(x,y)$, in the cut. Because the nodes are partitioned into two disjoint sets, a path from the source to the sink will include at least one arc in any cut. The maximum amount of flow that can travel through a cut is equal to the capacity of the cut; each cut capacity then is an upper bound on the value of the maximum flow.

The max flow - min cut theorem states that "the maximum flow between a source and a sink is equal to the minimum cut capacity of all cuts separating the source and the sink", [Ref. 9: p. 147]. The flow on each arc, $f(x,y)$,

in the minimum cut is equal to $c(x,y)$, the capacity of that arc. Additionally, if C and C' consist of all labelled and unlabelled nodes, respectively, then the set of arcs (C,C') identifies those arcs which comprise the minimum cut. If there is more than one minimum cut, the maximum flow algorithm identifies the cut closest to the source, [Ref. 7: p. 70].

It is now relatively easy to choose the 'best' arc for interdiction purposes. First, the effects of the obstacle must be modelled on each arc in the minimum cut set which the unit has the capability to interdict.

Let (w) represent the location of the force which will place the obstacle. If R represents the straight line distance between two points, then $R(x,w)$ can be thought of as the range from the emplacing force to the obstacle. Current doctrine requires that obstacles be covered by fire (indirect or direct) to enhance their capabilities. Therefore, these nodes which a unit can interdict satisfy the relation shown in equation 3.33, where \bar{R} and \underline{R} are allowable maximum and minimum ranges from the unit to the obstacle.

$$\underline{R} < R(x,w) < \bar{R} \quad \forall (x) \text{ where } (x) \in C \text{ or } C' \quad (\text{eqn 3.33})$$

Call the set of nodes identified by equation 3.33 S . Let S' be the set of arcs in the minimum cut which the friendly unit can interdict, as shown in equation 3.34.

$$S' = ((z,x) \text{ and } (x,y) | x \in S) \quad (\text{eqn 3.34})$$

The obstacle effects are input from the engineer module and include a reduced arc speed, $S(x,y)$, with which a reduced arc capacity, $c'(x,y)$, can be calculated. Flow on an arc in the minimum cut is equal to the arc capacity. If

the capacity of that arc is reduced, that arc is still in the minimum cut and flow on that arc is equal to the reduced arc capacity. The max flow-min cut theorem guarantees that maximum flow has been reduced by an amount equal to the reduction in arc capacity. The best arc to interdict is that arc which solves equation 3.35.

$$\max c(x,y) - c'(x,y) \quad \forall (x,y) \in S' \quad (\text{eqn 3.35})$$

The engineer module being developed will identify routines for allocating interdiction on multiple arcs.

2. Minimum Cost Route Selection

As previously stated, the network provides the structure on which unit movement can be simulated. Shortest path problems, which are a form of a minimum cost flow problem, can be used to determine minimum time or distance between two points and to select the best path to follow. The formulation of the shortest path problem presented here is in its canonical form [Ref. 6: p. 225]. In actuality, highly efficient labelling algorithms such as the Dijkstra and Ford algorithms, [Ref. 7: p. 47], are used to solve the shortest path problem and identify the shortest path.

In the shortest path problem, the goal is to find the least costly path of routing one unit of flow from the origin (s) to the terminal (t). This unit is required to stay intact, hence no capacities are modelled on the network. The cost of the path is the sum of the costs associated with each arc in the final path. If cost equals time, then this formulation is interested in finding the quickest time path (A) through the network.

a. Assumptions

- The unit moving in the network is represented as a point
- All time and distance costs are represented as arc attributes
- The network defined by all arcs (x,y) is a subnetwork of the larger, original network and represents the unit sector in which unit (k) can move
- The maximum arc speed, $S(x,y,k)$, is constant for each arc (x,y)

b. Notation

- $T(x,y,k)$ Time to travel the length of arc (x,y), hours
- $TP(A,k)$ Time for unit k to travel path (A), hours, where (A) is the shortest path from (s) to (t)
- $Z(x,y)$ Decision variable, $Z(x,y) = 1$ if arc (x,y) is in path (A), 0 otherwise
- n Total number of nodes in the subnetwork
- (s) Origin node, input from planning module
- (t) Destination node, input from planning module
- $L(x,y)$ Length of arc (x,y) input from data base
- $S(x,y,k)$ Maximum arc speed, computed from equation 3.2

c. Formulation

The time of travel on each arc (x,y) is shown in equation 3.36.

$$T(x,y,k) = L(x,y)/S(x,y,k) \quad (\text{eqn 3.36})$$

In the formulation shown below, $Z(x,y)$ represents the decision variable which determines if arc (x,y) is used in shortest path (A) for a single unit (k).

$$\min TP(A,k) = \sum T(x,y,k)Z(x,y) \quad \forall (x,y) \in \text{network} \quad (\text{eqn 3.37})$$

$$\sum Z(x,y) - \sum Z(j,x) \begin{cases} = 1 \text{ if } x=s \\ = 0 \text{ if } x \neq s \text{ or } t, \forall \text{ node } x \\ = -1 \text{ if } x=t \end{cases} \quad (\text{eqn 3.38})$$

$$Z(x,y) \in (0,1) \quad (\text{eqn 3.39})$$

d. Discussion

The formulation of the shortest path problem has little to do with the actual solution process. The Dijkstra algorithm uses a procedure of direct cost computation for the shortest path to each node called 'labelling'. A permanent label on a node indicates the time computed to reach that node is a minimum. A temporary label on a node indicates the time computed to reach that node is the least time found so far. The algorithm incrementally considers nodes for inclusion in the shortest path until the destination node (t) receives a permanent label. The steps in the Dijkstra algorithm and a heuristic proof that the result is a minimum can be found in [Ref. 7: p. 46].

The shortest path formulation presented above has assumed that only one unit was flowing in the network. The routing problem becomes much harder when attempting to find shortest paths with multiple units on the network. For example, a two unit, minimum cost problem requires two decision variables, one representing the allocation scheme of each unit. There is no known algorithm which allocates shortest path routings based on a combination of unit movement priorities, sequencing the use of arcs and nodes, and modelling congestion caused by slowed units. Such an algorithm, possibly using heuristic routines to limit the scope of the problem, may be developed for use in the research model in the future.

e. Interdicting the Shortest Path

Interdiction of the probable path of travel of an enemy unit at a point forward of its current location is a method that may result in decreasing that unit's inherent value. In this discussion, assume that the friendly commander is attempting to increase the arrival time of one enemy combat unit with a single obstacle that can be placed on an arc or at a node.

If the enemy unit's path is known, then obviously obstacle planning will concentrate on that path. If the enemy's path is unknown, and a probable destination can be identified, then a shortest path can be computed and obstacle planning considers only that path. There is a chance of selecting the wrong path, but interdicting the shortest path guarantees the enemy won't arrive in the quickest possible time.

To reduce the amount of computations required to evaluate the effects of the obstacle and avoid recalculating numerous shortest path problems, only those arcs and nodes on the enemy unit's path which can be interdicted by the friendly unit will be considered. Let S be the set of nodes which satisfy equation 3.40, where w is the friendly unit location, $R(x,w)$ is the straight line distance from node w to node x , and \underline{R} and \overline{R} are the minimum and maximum allowable ranges for an obstacle measured from w .

$$\underline{R} < R(x,w) < \overline{R} \quad \forall (x) \in \text{path}(A) \quad (\text{eqn 3.40})$$

The set S' is the set of arcs identified by equation 3.41.

$$S' = ((z,x) \text{ and } (x,y) | x \in S) \quad (\text{eqn 3.41})$$

The arc on which placing the obstacle achieves the greatest time delay satisfies the following formulation. Additional notation used in this formulation follows:

- $Ta(x)$ Time of arrival of the enemy unit at node (x) , computed during shortest path calculation
- $TIN(x,y)$ Time the interdiction of arc (x,y) could be complete, input from the engineer model
- $TDEL(x,y)$ Total delay time on arc (x,y) , decision variable computed by equation 3.42.

In the formulation, $TDEL(x,y)$ represents the decision variable which determines if arc (x,y) is the arc to be interdicted. (x,y) is directed from x to y .

$$\begin{aligned} \max TDEL(x,y) \quad & \forall (x,y) \in S' & (\text{eqn 3.42}) \\ \text{subject to} \end{aligned}$$

$$TIN(x,y) < Ta(x) \quad \forall (x,y) \in S' \quad (\text{eqn 3.43})$$

This problem could be solved quickly by enumerating all arcs contained in set S' . Because the obstacle effects will be aggregated into the existing arc attributes, and not modelled at a precise location, the time constraint must be based on the starting node of arc (x,y) .

3. Selection of a Unit's Future Location

The unit planning module will identify the criteria which cause a unit to displace to another location. The network provides the structure on which that unit can move, and through the use of location algorithms, the network can support the decision process of selecting a new location. Location algorithms are especially applicable to the CS/CSS units which relocate based on known locations of the supported units. However, location algorithms may not be appropriate when other factors such as survivability may be

of major concern. Location problems are divided into two broad categories, minisum and minimax problems. A minisum objective function seeks to minimize the average cost to all supported units, while a minimax objective function seeks to minimize the cost to the farthest supported unit. In the research model, cost will equal the shortest time between the CS/CSS unit and the unit receiving support.

The following formulation is a variation of what Handler calls the warehouse location problem, [Ref. 8: p. 59]. The decision variable Z_{ij} represents the fraction of support provided by a unit at node (i) to a unit at node (j) and solves for multiple unit locations.

a. Assumptions

- All units will be modelled as a point and located at nodes
- Location of units receiving support is known
- The network defined by all arcs (x,y) is a subnetwork of the larger, original network and represents the unit sector being modelled

Although combat units will rarely be located at a single node, the first assumption is realistic because of the dependence of the logistic functions on the transportation system and the identification of specific points such as aid stations, and unserviceable equipment collection points.

b. Notation

- (i) All nodes from which support may be given, $i=1...n$
- (j) All nodes which receive support, $j=1...J$, input from planning module
- P_j Weighting factor, priority of support to units (j), $0 \leq P_j \leq 1$, input from maintenance planning module

- $TP(i,j)$ Shortest time from (i) to (j), calculated using shortest path algorithm
- m Total number of support sites to be identified
- Z_{ij} Fraction of support supplied to (j) from site (i), $0 \leq Z_{ij} \leq 1$, decision variable

c. Formulation

In the following formulation, Z_{ij} represents the decision variable and identifies node locations for supporting units, (i), node locations of the supported units (j) and the fraction of support provided to each (j) by each (i).

$$\begin{aligned} &\text{minimize} \quad \sum_{i=1}^n \sum_{j=1}^J P_j \cdot TP(i,j) \cdot Z_{ij} \\ &\text{subject to} \end{aligned} \quad (\text{eqn 3.44})$$

$$\sum_{i=1}^n Z_{ij} = 1 \quad \forall j \quad (\text{eqn 3.45})$$

$$Z_{ii} \geq Z_{ij} \quad i=1 \dots n, j=1 \dots J, i=j \quad (\text{eqn 3.46})$$

$$\sum_i Z_{ii} = m \quad (\text{eqn 3.47})$$

$$1 \geq Z_{ij} \geq 0 \quad (\text{eqn 3.48})$$

Equation 3.45 insures that the unit at node (j) is fully supplied. Equation 3.46 insures that the unit at (j) can be supported only from nodes (i) at which a support unit is present; equation 3.47 restricts the number of selected nodes (i) to m.

d. Discussion

Due to the nature of military operations, combat units usually receive their support from a single supporting unit, each responsible for a separate class of supply. For

example, an armor battalion would receive ammunition from one ammunition supply point and get its vehicles repaired by one maintenance company. (Identification of these support relationships will be discussed in the next chapter). This restriction causes equation 3.49 to replace equation 3.48 in the formulation.

$$Z_{ij} \in (0,1) \quad i=1\dots n, \quad j=1\dots J \quad (\text{eqn 3.49})$$

Additionally, support units are usually restricted to the sector of the combat unit to which they alone provide support. For example, a maintenance company supporting the three maneuver battalions of a brigade will be restricted to that part of the brigade sector known as the 'brigade rear', i.e. forward of the brigade rear boundary and behind the battalion rear boundaries. This further reduces the formulation to finding the best location (i) for a single supporting unit. The constraints to the formulation then become routine checks on the data for consistency.

Handler discusses the difficulty of solving this formulation and states most algorithms "...may be roughly classified into two groups, those that use branch and bound techniques...and those that start with solving a linear programming relaxation...." [Ref. 8: p. 60].

H. DISCRETE SIMULATION OF THE EXECUTION NETWORK

Previous sections have discussed maximum flow and shortest path problems and introduced techniques to find the arc or node where interdiction would cause the largest disruption in flow or time delay. This section will discuss interdicting units on the execution network in an attempt to both attrite the enemy force and create an obstacle at the

interdiction site. A discrete network simulation is key in this process because it:

- models the location of the unit being interdicted as a line segment
- models unit dispersion and formations which can be used in routines which assess damage
- models off road mobility factors which allow the attacked unit to bypass destroyed vehicles or congested areas
- provides the structure to compute shortest paths to predicted locations, and arrival times at all nodes of the shortest path.

The formulation which follows is just one model which may be used to allocate interdiction resources against enemy units moving in the network. Research is currently being directed at many of the functions necessary to support this formulation. Developing a function is very difficult due to the dependence of several variables on one another and efforts are on-going to identify these relationships.

a. Assumptions

- Unit (k), which may be interdicted, is modelled as a line segment
- Intelligence planning modules can supply the probable destination node (t) of unit (k)
- If the path of the unit being considered for interdiction is not known, that unit is considered moving on the shortest path to some objective input from the intelligence planning module.
- Each unit (k) can be attacked exactly once.
- The effects of an attack on unit (k) have no influence on the current state of all other units.
- An attrition module exists which can predict the results of an attack against unit (k)

- A value function exists which can calculate a unit's current inherent value and future worth.
- A time delay function exists which can calculate the additional time effects of an obstacle on an arc or node
- Interdiction is a homogeneous entity which can be allocated in unequal portions
- A weighting function exists which allocates interdiction between attrition effects and delay effects

b. Notation

- i Index of arc or node in a specific shortest path, $i=1\dots n$
- j Index of desired effect of interdiction, $1=\text{attrition}$, $2=\text{delay}$
- k Index of unit which is attacked, $k=1\dots K$
- W_{jk} Weight of desired effect j on unit k , $0 \leq W_{jk} \leq 1$, input from the planning module
- Z_{ik} Decision variable, 1 if unit k attacked on arc or node i , 0 otherwise
- X_k Decision variable, amount of interdiction allocated to attack unit k
- \bar{X} Total amount of interdiction available, input from engineer planning module
- VR_{ik} Reduced value of unit k following interdiction on i , calculated in equation 3.50
- TD_{ik} Time delay incurred by unit k following interdiction on i , calculated in equation 3.51
- TRV Total reduced value, desired output

Unit attributes previously defined:

- $NVEH(j,k)$ Number of vehicles of type j in unit k
- $Thu(k)$ Halted unit time delay

Obstacle attributes previously defined

- T_d Time delay of an obstacle

- Tb Breach time of an obstacle

Arc and node attributes previously defined:

- SURF Arc surface
- MC Mobility class
- ORMP Off road mobility pointer
- ALTRTE Alternate route pointer
- NLANES Number of lanes for an arc

c. Formulation

A functional relationship for a unit's reduced value following an attack is shown in equation 3.50.

$$VR_{ik} = f_8(W_{lk} X_k, NVEH(j, k)) Z_{ik} \quad (\text{eqn 3.50})$$

A similar functional relationship for a unit's additional delay time to reach its objective following an attack on that unit is shown in equation 3.51.

$$TD_{ik} = f_9(W_{2j} X_k, SURF, MC, ORMP, Thu, Tb, Td, NLANES) Z_{ik} \quad (\text{eqn 3.51})$$

In the following formulation, Z_{ik} represents the decision variable which indicates if arc or node i is used in the final solution. X_k is the decision variable which represents the amount of interdiction allocated against unit k .

$$\begin{aligned} \max TRV = & \sum_i \sum_k f_{10}(VR_{ik}, TD_{ik}) \\ \text{Subject to:} & \end{aligned} \quad (\text{eqn 3.52})$$

$$\sum_j W_{jk} = 1 \quad \forall k \quad (\text{eqn 3.53})$$

$$\sum_k X_k \leq X \quad (\text{eqn 3.54})$$

$$\sum_i Z_{ik} = 1 \quad \forall k \quad (\text{eqn 3.55})$$

d. Discussion

The formulation, as it currently stands, is a resource allocation problem with a goal of reducing an enemy unit's value by the greatest possible amount. It is a difficult problem to solve due to the presence of two decision variables and the prospect of having to use enumeration along the shortest paths of multiple units. Methods introduced earlier can be used to identify those nodes and arcs which the attacking unit can range with its interdiction assets, reducing the set of possible nodes and arcs, i.

Additional constraints are added to the problem when time factors are considered. For example, the attacking unit may have a time 'window' in which the interdiction must occur. An artillery unit which is currently moving must take certain actions before it can interdict an enemy unit. Other units have maximum times they can remain in one position before they must relocate to avoid being interdicted themselves. In order for interdiction to be effective in this problem, both it (the interdiction) and the enemy unit must arrive at the same place at the same time.

The formulation is unrealistic because of the assumed independence of attrition effects between units. In actuality, non-targeted units may also be attrited because of their proximity to a unit being attacked. Collateral damage from a nuclear blast could affect several units at once. In addition, the delay of one unit will have a significant effect on the arrival times of numerous other units. The damaged vehicles of the attacked unit form an obstacle which delays any other units scheduled to use that portion of the network. If the attacked unit is forced to stop its movement, then its presence also causes additional delays to follow-on units using that portion of the network.

A discrete movement simulation will be required to step units through the network. Each step will be required to evaluate a unit's value based on projected collateral damage effects and changes to the network along that unit's projected path. Construction of an algorithm to solve such a problem will be very difficult and enumeration of the changes in every unit's value at every step will be prohibitive. Heuristic decision rules will be necessary in concert with current doctrine to eliminate interdiction possibilities not within the projected capabilities or plans of a unit.

Another constraint is added to the formulation when the interdiction assets are modelled as discrete weapons systems and not a homogeneous resource capable of being apportioned to a unit. Some weapons are often designed for a specific purpose while others are employed together to enhance their effects.

As additional constraints are identified, the solution to the formulation presented above becomes more difficult to attain. Decision and allocation rules will be necessary to reduce the scope of the problem such that predictions can be based on feasible solutions rather than attempting to find an optimal solution.

IV. DEVELOPMENT OF THE COMMAND AND CONTROL CONNECTIVITY NETWORK

A. BACKGROUND

This chapter will describe in detail the components and functions of a command and control (C²) connectivity network. The chapter's main emphasis will be directed at identifying problems inherent in supporting the research model's goal of variable resolution and presenting network attributes which support varying resolution schemes.

If one assumes that each unit is a self contained 'combat packet' (i.e. capable of fighting and sustaining itself without any outside help) and each unit reports solely to its immediate superior, then the flow of information in a network is trivial to model. These 'combat packets' rarely occur as force structure varies with each operation. One purpose of the C² connectivity network model is to provide a framework that can be used to investigate the effects of a changing force structure on information flow.

The C² network will usually be structured as a tree, as presented in Chapter I. Specifically, the C² connectivity network will provide the structure to:

- Monitor information flow around the network
- Initiate events in the execution modules
- Assess the quality of results of command and control planning
- Investigate the details of the internal C² process

A message of some sort serves as the only input to a unit's processing algorithm. This message initiates actions within the receiving unit's planning module and often

results in a new message being transmitted. Units are often required to change the destination of their messages as the force structure changes with each operation. In the model, changes to the force structure will be reported by changing the connectivity of the task force organization. This requires that processing algorithms for information be linked to routing algorithms. The processor will functionally identify the destination of a message (i.e. superior unit responsible for logistic requests) while the connectivity network identifies the destination by unit name. The force structure may be reconfigured without changing the structure of the processing algorithms. Additionally, the C² network will provide information about time delays and congestion for each of the modes of transmission.

In some low resolution C² models, information will be required to flow between units in order to initiate events. In these models, a detailed description of the C² process is not required. Information is passed solely to cause events to occur. As the resolution of the model increases, more care must be focused on the quality of decisions made in the C² process.

At the highest level of resolution, one is interested in the details of the planning process and how decisions are made. A high resolution model must represent staff elements explicitly following a detailed analysis of tasks conducted by each staff section.

A C² network will be extremely difficult to develop in its final form, and it is comprised of two disjoint processes.

- First, the process of identifying who sends messages to whom, where do these messages go and how do they get there.
- Second, the process of what happens to that message once it is received by a unit, i.e. information processing.

Little is currently known about the inner workings of information processing within a unit or staff section. A large portion of the research effort is being directed towards identifying the critical decision criteria of information processing.

The goals of this chapter, which center primarily around the first of the two processes, are as follows:

- Identify key arc, node and message attributes to form the basic structure of the network. The attributes may be further subdivided, dependent on the level of resolution being modelled.
- Discuss the problems associated with varying force structure and the flow of different types of information.
- Provide the structure of how the attributes are used in a discrete event simulation.
- Provide the structure to determine appropriate rate and capacity constraints and determine if a message can be transmitted.

The processing algorithm for each unit is a subject that will not be discussed in great detail. In general terms, however, processing algorithms must conduct several basic functions. Initially, incoming messages must be partitioned into the appropriate sections representative of jobs within the processor. Within each section, internal processing results in some decision being made which may trigger further additional actions on the part of the processor. Some information is intended merely to update current information known about friendly or enemy units. Other types of information are processed and then sent unchanged as output to other units. The content of some messages is changed and the processor output is a new message structured as either an order, request, or information.

Before the nodes and arcs can be described, one must understand the four types of information that will flow in the C^2 network and the effects of task organizing on the direction of that flow.

B. INFORMATION AND TASK ORGANIZATION

1. Types of Information

Within the research model, information will be classified as one of four types introduced in Chapter II: orders, requests, reports, or intelligence.

Orders are those instructions passed from superior to subordinate units which must be executed by the subordinate unit, to include those normally associated with advisory channels such as logistics and maintenance. In planning modules within a unit headquarters, feasibility checks will be accomplished prior to orders being issued. Tentative orders from superiors will be used as the basis for feasibility checks by subordinates. In execution modules, orders will initiate actions by the subordinate (receiving) unit; first, a feasibility check by that subordinate unit, and second, initiation of its own planning sequence.

Requests are information passed from subordinate to superior; the subordinate unit is asking either for approval of a planned, feasible course of action or for a reallocation of support, such as artillery support, which the superior unit controls. Requests will require action on the part of the superior unit; first a feasibility check within its overall plan and second, an answer in the form of an order to the subordinate.

Reports are information passed between units concerning friendly units. Reports update current information available to a unit. Reports require no further action

between units, however some action may be required within the receiving headquarters.

Intelligence is information passed between units concerning enemy units. Intelligence, like friendly unit reports, serves to update current information available to a unit.

Additionally, information falls in one of two general categories based on the nature of the message: tactical or logistic. Information in the tactical category, which includes all four types of information, is usually considered the information required to fight the force. Information in the logistic category includes only orders, requests, and reports, and is usually thought of as the information required to maintain and sustain the fighting force. In total, there are seven feasible type and category combinations.

2. Effects of Task Organization

a. Definitions

A task organization is defined in the Staff Officers Handbook, RB 101-999 (T), as "a temporary grouping of forces designed to accomplish a particular mission. Task organization includes the distribution of available assets to subordinate control headquarters by attachment or by placing assets in direct support or under the operational control of the subordinate", [Ref. 11: p. B-29].

In short, task organizing is a method of tailoring a fighting force for a specific mission. This practice is often used by the U.S. commander to allocate the entire spectrum of forces available to him. The Soviet commander does not cross-attach his units as frequently as his U.S. counterpart. Even then, Soviet units assigned to team with another unit appear to remain under some measure

of control of their parent unit; much like a 'direct support' (U.S) role, [Ref. 12: p. 3-69].

Current U.S. doctrine states that a division headquarters (or lower) allocates the combat forces two levels beneath it to support the tactical course of action. Based on this force dispersion, the superior unit then allocates the control headquarters one level beneath it. CS and CSS assets are allocated as necessary. Table V presents a typical allocation scheme; specific numbers and types of units depend on the current mission.

TABLE V
Typical Unit Allocation Scheme

Tactical HQ	Allocates	To
Corps	Corps CS/CSS Assets	Divisions
Division	Battalions Division CS/CSS Assets	Brigades
Brigade	Companies	Battalions

The manner of control of subordinate units (attached versus operational control for example) is the factor that determines the direction and type of information flow for each category of information. Control is further subdivided into two main categories: command relationships and tactical missions. The definitions which follow are general in nature; specific applications are defined by each combat or support function. Each of the terms introduced below is defined in Appendix A, Operational Terms and Definitions.

Command relationships specify command channels; they are usually established because of special operational capabilities or unique logistic requirements of the subordinate unit. A unit can have one of three command relationships with a superior headquarters; that subordinate unit is said to be either

- Organic,
- Attached, or
- Operationally controlled

An organic relationship implies a unit receives both command and logistics from its parent unit. A unit is assumed organic to its immediate superior, unless otherwise specified .

A unit which is attached receives its command and most of its logistics from the superior unit to which it is attached. Personnel replacement for an attached unit is handled by that unit's parent unit, (i.e. the unit to which it is organic).

A unit which is operationally controlled (OPCON) receives its orders from the unit to which it is OPCON and receives its logistic support from its parent unit.

Table VI summarizes the identity of the superior unit which will receive information in the model for a given command relationship.

Support units such as artillery and engineers usually receive tactical missions in lieu of command relationships. Support units can have one of four tactical missions, listed below, which define their responsibilities to superior headquarters.

- Direct support (DS)
- General support (GS)
- Reinforcing (R)
- General support (reinforcing) (GSR)

TABLE VI
Command Relationship Information Flow

Command Relationship	Information Type	
	Tactical	Logistic
Organic	Parent Unit	Parent Unit
Attached	Unit to which attached	Unit to which attached
OPCON	Unit to which attached	Parent Unit

A unit in direct support of another is required to give its priority of support to the supported unit.

A unit in general support provides support to the force as a whole, not a particular subdivision, and is commanded by its parent unit.

Reinforcement and general supporting (reinforcing) are artillery missions which define an artillery unit's relationship to another artillery unit.

b. Connectivity and Information Flow

The destination of information flow between units is defined once a commander has allocated his forces, delineated command relationships for those units cross-attached, and assigned tactical missions as appropriate. Command relationships may actually require a unit to report to two controlling headquarters; one for operational purposes, the other for logistic purposes.

Combat maneuver units, such as battalions, are usually task organized to brigades as either attached or OPCON units. Fire support units generally receive tactical

missions and remain under the command of their parent unit. Figure 4.1 shows the responsibilities of air defense artillery units for information flow, from which one can infer several specific links in the connectivity network. Figure 4.2 presents the same information for field artillery units.

[Ref. 13]

	GENERAL SUPPORT (GS)	GENERAL SUPPORT REINFORCING (GS R)	REINFORCING (R)	DIRECT SUPPORT (DS)
Who establishes AD priorities?	The force commander.	(1) The force commander (2) The supported commander through the reinforced ADA commander	The supported commander through the reinforced ADA commander.	The supported commander.
Who locates the ADA unit? ¹	The commander assigning the mission in coordination with the supported force ground commander	The commander assigning the mission in coordination with the supported force ground commander	The reinforced ADA commander in coordination with the supported force ground commander	The DS ADA commander with approval of the local ground commander
Who positions ADA fire units? ²	ADA fire unit commanders in coordination with the local ground commander.	ADA fire unit commanders in coordination with the reinforced ADA unit commander and the local ground commander	ADA fire unit commanders with approval of the reinforced ADA unit commander and the local ground commander	ADA fire unit commanders with approval of the local ground commander.
With whom is liaison established?	As required	As required but including the reinforced ADA commander	As required but including the reinforced ADA commander	Supported unit commander
With whom are communications established?	As required	As required but including the reinforced ADA unit	As required but including the reinforced ADA unit	Supported unit
What is the mode of control?	(1) Centralized. (2) Decentralized	(1) Centralized (2) Decentralized	(1) Decentralized (2) Centralized	Decentralized

¹The terms "locates" and "locating" specify the establishment of a broad operating area (commonly, a "goose egg").

²The terms "positions" and "positioning" specify the selection of an exact point within the operating area. (Although not addressed here, the terms, "sites" and "siting" specify the placement of the individual items of equipment on selected spots within the position.)

Figure 4.1 Air Defense Artillery Support Relationships

CS and CSS units are generally allocated from a combined list of command relationships (attached, OPCON) and tactical missions (DS, GS). Figures 4.3 and 4.4 specify the responsibilities for information flow for aviation and engineer units, respectively.

[Ref. 13]

INHERENT RESPONSIBILITIES

An FA unit with a mission of—	Answers calls for fire in priority from—	Furnishes (FSO/LO)—	Establishes communications with—	Has as its zone of fire—	Furnishes fire support team	Is positioned by—	Has its fires planned by—
General support (GS).	1. Force FA HQ. 2. Own observers.	No requirement.	No requirement.	Zone of action of supported unit.	No requirement.	Force FA HQ.	Force FA HQ.
General support-reinforcing (GSR).	1. Force FA HQ 2. Reinforced unit. 3. Own observers.	LO to reinforced FA unit HQ.	Reinforced FA unit HQ.	Zone of action of supported unit to include zone of fire of reinforced FA unit.	No requirement.	Force FA HQ or reinforced FA unit, if approved by force FA HQ.	Force FA HQ.
Reinforcing.	1. Reinforced FA 2. Own observers. ¹ 3. Force FA HQ	LO to reinforced FA unit HQ.	Reinforced FA unit HQ.	Zone of fire of reinforced FA unit.	No requirement.	Reinforced FA unit or as ordered by FA HQ.	Reinforced FA unit HQ.
Direct support (DS).	1. Supported unit. 2. Own observers. 3. Force FA HQ	FSO to each maneuver battalion and brigade of the supported unit.	FIST chiefs and FSO of supported maneuver units.	Zone of action of supported unit.	To each maneuver company.	DS FA unit commander or as ordered by force FA HQ.	Develops own fire plans.

¹ Includes all target acquisition means not deployed w/supported unit (radar, aerial observation, survey parties, etc.)

TACTICAL MISSIONS

Figure 4.2 Artillery Support Relationships

[Ref. 13]

		An aviation unit with a tactical mission status of—	Receives missions and tasks from—	Is under command and control of—	Establishes liaison with—	Is task organized by—	Receives combat service support through—	Can be given further status/ tactical mission of—
TACTICAL MISSION	General support	Ground units supported with priorities assigned by ground unit receiving GS	Higher aviation unit commander.	As directed by HQ receiving GS.	Aviation unit commander.	Normal aviation unit CSS channels	NA	
	Direct support.	Ground unit supported.	Higher aviation unit commander.	Unit being supported	Aviation unit commander.	Normal aviation unit CSS channels	NA	
	OPCON.	Unit to which OPCON.	Unit commander to which OPCON.	As directed by HQ exercising OPCON	Unit commander to which OPCON	Normal aviation unit CSS channels	OPCON DS GS	
	Attachment.	Unit to which attached	Unit commander to which attached.	As directed by HQ to which attached.	Unit commander to which attached	Unit to which attached, unless otherwise stated in attachment order.	Attachment OPCON DS GS	
COMMAND RELATIONSHIPS								

Attack helicopters and air cavalry organizations are aerial maneuver units and are assigned only command relationships. Other aviation units are assigned tactical missions.

Figure 4.3 Aviation Command and Support Relationships

[Ref. 13]

An Engineer element with a relationship of—	Is under command and control of—	Establishes liaison and communication with—	May be task organized by—	Relationship/assignment normally given—	Responds to support requests from—	Work priority established by—	Spare work effort available to—	Request for additional support forwarded through—
Attached	Supported unit	Supporting to supporting unit	Supported unit	Further attached or DS to brigades or task forces	Supported unit	Supported unit	Supported unit	Supported unit
OFCON	Supported unit, less administration/logistics	Supporting to supported unit	Supported unit	Further OFCON to other engineer maneuver units or DS to brigades or IFs	Supported unit	Supported unit	Supported unit	Supported unit
General support (GS)	Parent unit	Supporting to supported unit	Parent or supporting unit	Support of a force as a whole & employed in a brigade/division rear area by area/task assignment	Parent unit	Parent unit	Parent unit	Parent unit
Direct support (DS)	Parent unit	Supporting to supported unit	Supporting unit	Support of a particular size unit through task or area assignments	Supported unit	Supported unit	Parent unit	Parent unit

Figure 4.4 Engineer Command and Support Relationships

C. FORMULATION OF THE C² NETWORK

The complexity of any command and control model is directly related to the level of resolution being utilized. The objectives of the C² network, at any level of resolution, will be to provide a structure which:

- Determines the current status and remaining capacity at both nodes and arcs
- Determines if a unit j , desiring to send a message of type i to unit k via mode of transmission l can actually do so
- Allows the formation of queues of backlogged messages

The formulation of the network to be presented in this section will address the first two objectives.

1. Subscript Notation

The following notation will be used in this chapter unless specifically stated otherwise:

- i ; Category and type identifier of a message, $i=1...7$
- j ; Unit designation of a unit sending a message
- k ; Unit designation of a unit receiving a message
- m ; Unit designation, unspecified as to either sending or receiving unit
- l ; Transmission mode used to transmit a message, $l=1...3$

The subscript i , ranging from 1 to 7, represents the seven possible combinations of message categories and types. Message categories and types were discussed in the previous section; the possible combinations are summarized in Table VII.

The subscript l , ranging from 1 to 3, represents the possible modes of transmission and are summarized in Table VIII. In general, the radio and wire modes will be used to transmit messages between superior and subordinate units,

TABLE VII
Summary of Message Category-Type Combinations

CATEGORY	TYPE	INDEX, i
Tactical	Order	1
	Request	2
	Report	3
	Intelligence	4
Logistic	Order	5
	Request	6
	Report	7

which in actuality are often separated by great distances. The voice mode will primarily be used between staff sections of a single headquarters when each section is modelled as a separate unit.

TABLE VIII
Summary of Modes of Transmission

MODE	INDEX 1
Radio	1
Wire	2
Voice	3

2. Resolution Issues

The network presented in this section can be thought of as a 'mid-resolution' network. The nodes will represent any unit or staff section for which a separate processor is developed. For the rest of this chapter, the term 'unit'

will be representative of any headquarters or staff section which is modelled as a unique node. An arc will represent a transmission mode, *l*, existing between two units.

The modeler still has the option to define message category and type, *i*, and transmission mode, *l*, to fit the needs of the particular study. For example, message category and type could be aggregated into just two combinations, one based on tactical messages, and the other based on logistic messages. In a high resolution model, the radio mode could be disaggregated into the several frequencies normally assigned to a unit such as a brigade operations section.

The parameters of *i* and *l* must be fully defined before message routing algorithms, which will attempt to transmit a message of type *i* via mode *l*, can be finalized. Routing algorithms will be required to consider the nature of a message and prioritize acceptable modes of transmission, possible time delays and message priority.

A fourth mode of transmission, the courier, commonly used in actual practice, was not included in this formulation for several reasons. First, the regularly scheduled courier carries messages which, by their nature, are only transmitted by courier. This includes routine status reports and intelligence summaries of little immediate tactical value. Second, the courier is (infrequently) used between subordinate and superior units when the primary means of transmitting urgent messages (radio, wire) are either electronically jammed or non-operational. Third, the model is interested in those messages which cause a unit to react or begin planning actions which result in additional actions or messages being generated. Courier transmitted messages often arrive at a unit in batches, form the backlog queues, and are only acted on when a unit is processing no other messages.

Reliability of an arc, R_e , to be measured in percent of time available, is partly based on the proximity of the sender-receiver pair and the mode of transmission. Arc reliability for a specific mode will be based on mode characteristics such as the failure rate distribution of the device and its repair time distribution. This formulation will represent reliability as the probability that an arc is operational at any point in time; prior to using an arc, operability will be checked using a Monte Carlo process. Reliability as it regards the research model has been previously discussed in [Ref. 5]. A more explicit representation to determine arc operability would model failure rates and repair times individually.

Finally, this formulation will only address a single superior-subordinate pair, (i.e. two levels of the hierarchy found in the force structure) and lateral communications flow between units all of which comprise one headquarters. In a more involved model representing three or more level of the hierarchy an additional set of filters would be needed to functionally identify the relation of unit j to unit k as one of three 'directions'. These three directions are 'up' the hierarchy to a superior unit, 'down' the hierarchy to a subordinate unit, and 'laterally' across a hierarchy to a sister unit. The filters would be needed due to the practice of dedicating certain transmission modes or devices to specific (j,k) pairs.

3. Data Base Attributes

The following discussion will identify those characteristics of arcs, nodes, and messages which will form the data base of the C^2 network. Each attribute is introduced by first using its notation, and is followed by a short description and its units, if applicable.

a. Node Attributes

- m; Unit descriptor
 - ND(m,1); Number of devices at unit m for either receiving or transmitting messages using transmission mode 1
 - TCAP(m,1); Total capacity of unit m to send or receive messages via mode 1, measured in characters per second
- TCAP(m,1) is calculated as shown in equation 4.1; RT(1) represents the arc transmission rate, measured in characters per second.

$$TCAP(m,1) = ND(m,1) \times RT(1) \quad (\text{eqn 4.1})$$

b. Arc Attributes

- l; Transmission mode, $l=1\dots3$
- Z(j,k,l); Number of possible arcs of type l between unit j and unit k
- RT(1); Transmission rate of one device of type 1, measured in characters per second
- CAP(j,k,l); Total capacity from unit j to unit k via mode l, measured in characters per second
- Re; Reliability, measured in percent
-

$$W(i,j,k,l) \begin{cases} = 1 & \text{if message type } i \text{ is doctrinally} \\ & \text{allowed to be transmitted} \\ & \text{from unit } j \text{ to unit } k, \text{ via mode } l \\ = 0 & \text{otherwise} \end{cases}$$

Z(j,k,l) is calculated using equation 4.2, which leads to the restriction in equation 4.3.

$$Z(j,k,l) = \min(ND(j,l), ND(k,l)) \quad (\text{eqn 4.2})$$

Arc capacity is found using equation 4.4.

$$Z(j,k,1) = Z(k,j,1) \quad (\text{eqn 4.3})$$

$$\text{CAP}(j,k,1) = \text{RT}(1) \times Z(j,k,1) \quad (\text{eqn 4.4})$$

c. Message Attributes

- i ; Message type, $i=1 \dots 7$
- $\text{LH}(i,1)$; Length of message type i when sent via mode 1, measured in characters
- $\text{TTM}(i,1)$; Time of transmission of message type i using mode 1, measured in seconds

LH is a function of 1 because the unit of measure could differ for various transmission modes; e.g. lines/minute or bits/second. However, the units of $\text{LH}(i,1)$ and $\text{RT}(1)$ must be consistent.

$\text{TTM}(i,1)$ is calculated using equation 4.5.

$$\text{TTM}(i,1) = \text{LH}(i,1) / \text{RT}(1) \quad (\text{eqn 4.5})$$

4. Current State Attributes

The following attributes are required to describe the current state of nodes and arcs.

a. Node Attributes

- $\text{NDU}(m,1)$; Number of devices of mode 1 currently in use at unit m
- $\text{RN}(m,1)$; Current transmission rate of all messages transmitted via mode 1, either into or out of unit m , measured in characters/second

b. Arc Attributes

- $\text{M}(i,j,k,1)$; Number of messages of type i currently being sent from unit j to unit k via mode 1

- $RC(j,k,l)$; Current transmission rate of messages from unit j to unit k via mode l , measured in characters/second

The number of devices of mode l currently in use at unit m , either receiving or transmitting messages, is computed using equation 4.6. Equation 4.6 assumes that each device can only be used to send or receive one message at a time.

$$NDU(m,l) = \sum_i \sum_k M(i,m,k,l) + \sum_i \sum_j M(i,j,m,l) \quad (\text{eqn 4.6})$$

The current rate of transmission for both arcs and nodes are determined as follows. Current node transmission rate for a unit, m , must account for all messages simultaneously entering and leaving the node, as shown in equation 4.7.

$$RN(m,l) = \sum_i \sum_k M(i,m,k,l) \cdot RT(l) + \sum_i \sum_j M(i,j,m,l) \cdot RT(l) \quad (\text{eqn 4.7})$$

The current arc transmission rate must account for all messages simultaneously flowing from unit j to unit k and from unit k to unit j , as shown in equation 4.8.

$$RC(j,k,l) = \sum_i M(i,j,k,l) \cdot RT(l) + \sum_i M(i,k,j,l) \cdot RT(l) \quad (\text{eqn 4.8})$$

5. Discrete Event Simulations

A unit processor for unit j identifies the need to transmit a message of type i to unit k , using mode of transmission l . Before that message can enter the C^2 network, certain constraints pertaining to node or arc availability must be satisfied. The current transmission rates and the number of devices used in the discrete event simulation form the basis of the checks. The main function of the discrete

event simulation is to update the value of $M(i,j,k,1)$ by use of $TTM(i,1)$.

The following example demonstrates the use of the discrete event simulation and the constraints to be introduced. Figure 4.5 shows that an arc of mode 1 exists between unit j and its subordinate unit k . Both j and k have two devices of type 1 currently operational; thus $Z(j,k,1)=2$. The rate of transmission of a device is fixed at 10 characters/second. The maximum node transmission rate, $TCAP(m,1)$, therefore is equal to 20 characters/second and the arc maximum rate of transmission, $CAP(j,k,1)$, is also 20 characters/second.

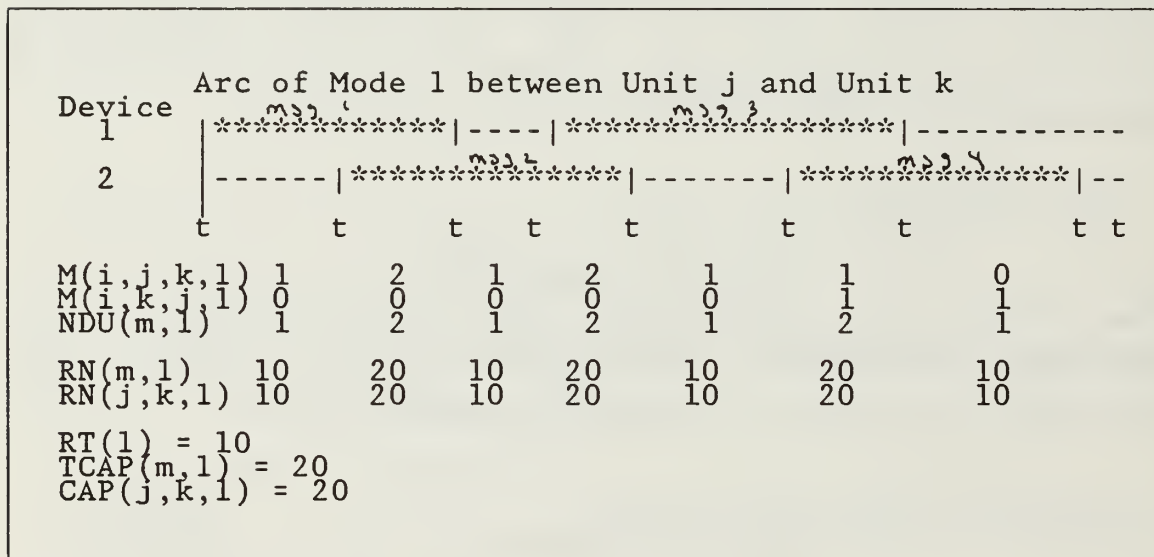


Figure 4.5 Discrete Event Simulation Example

In this example, the first three messages are sent from unit j to unit k . The last message is transmitted from unit k to unit j . The data below the arc, and between the designated times (such as t_1 and t_2), represents the state of the arc at that time.

Two checks must be performed at both the transmitting and receiving nodes to check for node availability. Equation 4.9 checks the number of devices of mode 1 for availability. Equation 4.10 insures that the current node transmission rate is less than its maximum transmission rate.

$$NDU(m,1) + 1 \leq ND(m,1) \quad (\text{eqn 4.9})$$

$$RN(m,1) + RT(1) \leq TCAP(m,1) \quad (\text{eqn 4.10})$$

If both equations are satisfied for m equal to both j and k, then the message of type i can leave node j and enter node k via mode 1.

It is recognized that the node capacity constraint in this formulation is a redundant constraint on that check to be presented for the arc. However, equation 4.10 could be required for certain devices which handle multiple messages such as multichannel communications equipment and field telephone switchboards.

Two checks are also needed to insure arc availability. Equation 4.11 is a compatibility check that message type i is doctrinally allowed to flow from unit j to unit k via mode 1. Equation 4.12 insures that the current arc transmission rate is less than its maximum transmission rate.

$$W(i,j,k,1) = 1 \quad (\text{eqn 4.11})$$

$$RC(j,k,1) + RT(1) \leq CAP(j,k,1) \quad (\text{eqn 4.12})$$

Again, if both arc constraints are satisfied, the arc formed by mode 1 is available to carry a message of type i from unit j to unit k. All four checks must be satisfied in

order for a message of type i to be transmitted on the network.

In this example, only one unit was modelled subordinate to unit j . Suppose now that there are multiple units, k_1 and k_2 , subordinate to unit j , all with two devices of mode 1. The constraints shown in equation 4.9 for the number of devices accounts for the situation where unit j communicates with units k_1 and k_2 on the same radio frequency and the use of the link by (j,k_1) prohibits the use of the link by (j,k_2) . In this example, let both devices in figure 4.5 be in use by (j,k_1) . A message of type i from unit j to unit k_2 which satisfies equations 4.11 and 4.12 could not be sent because of the device constraint.

The attributes of the nodes, arcs, and messages have been developed to support various levels of resolution. The above example could be expanded (or aggregated) to explicitly model additional (fewer) modes of transmission, 1, individual frequencies within a certain mode 1, or several additional (fewer) message category and type combinations. The modeler still has the option to define units j and k as large or as small as the needs of his study dictates.

V. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

This report has begun the development of one of the major modelling topics being researched for use in the Airland Research Model and has shown that a general network methodology is a feasible process for representing the interdiction battle in the corps rear area.

The features inherent in two combat processes, i.e. the underlying transportation system, composed of an execution network and multiple planning networks, and command and control systems were exploited to develop a viable network representation of each structure. Attributes were defined for the arcs and nodes in such a manner that network properties could be determined and the flow or movement of information or units could be modelled.

In Chapter III, both the execution and planning networks were developed. Attributes for use on nodes and arcs in the execution network were presented and modified for use in planning networks. Methods of representing obstacles in the network and evaluating their effect on shorest path times were also presented. These methods relied on re-evaluating only the portion of the path affected by insertion of an obstacle, and not a re-enumeration of an entire path or paths. The formulations presented at the conclusion of Chapter III show that exploiting the mathematical structure inherent in network models is a viable process for representing movement in a combat model.

The discussion in Chapter IV centered around the development of a discrete network used to conduct a discrete event simulation. Resolution issues and a discussion of

message destination based on a changing task force command relationship were introduced. The network presented in Chapter IV was comprised of arcs and nodes which were described by attributes supportive of a varying level of resolution.

Attainment of the goals of this thesis, as they were listed in Chapter I, is summarized in the following discussion. In general, the goals were developed in greater detail for the transportation network than the command and control interconnectivity network because so little is known about the decision process that takes place in each staff planning section.

Economy of representation can be achieved in the research model, dependent on the level of resolution being studied. The amount of significant terrain features in the transportation network and unit representation in the C² network can be controlled for each level of resolution. The level of resolution can vary between specific portions of the battlefield depending on the need for an explicit representation of a specific process.

Specific algorithms for use in the transportation network, such as maximum flow, shortest path, and location problems were identified. The transportation network, as it currently stands can be used to simulate the movement of multiple units and locate each in space and time.

The attributes of the arcs and nodes were shown to be supportive of models using various levels of resolution and adaptable for use in both continuous flow models and discrete movement simulations.

B. COMPARISON OF FLOW MODELS AND DISCRETE SIMULATIONS

When compared to the goals of this thesis, the following advantages and disadvantages of using network flow models on

the planning network and discrete simulations on the execution network can be drawn.

1. Use of Continuous Flow Models

Network flow algorithms are appropriate for use in low resolution representations of the flow of combat forces and logistic products. Specifically, formulation of a maximum flow problem allows one to model the arrival of combat forces at various parts of the battlefield over long periods of time. The shortest path algorithm can be used to determine the most likely path of a single combat unit.

The effects of obstacles and halted units to movement on the network can be easily accounted for by representing these effects in the attributes of the arcs and nodes.

Planning for network interdiction to disrupt maximum flow can exploit the maximum flow-minimum cut theory to identify optimal locations for obstacles.

Identification of the optimal location for an obstacle when interdicting a unit's possible path can be accomplished by measuring the effects of an obstacle along that unit's path. By enumerating the effects of the obstacle on a small number of arcs relative to the overall number of arcs with the unit's sector, the arc on which an obstacle delivers the maximum effects can be identified.

Several well known, computationally efficient algorithms are known which can be used to solve the maximum flow and shortest path problems.

Combat units in continuous flow models are assumed to be homogeneous entities, preventing the interdiction of specific units to support specific operational goals.

2. Use of Discrete Network Simulations

Use of a discrete simulation provides an explicit high resolution representation of a single unit as it moves through the execution network. The discrete simulation will be required to model interdiction against that unit to fully appreciate the effects of unit attrition and network delay.

A discrete simulation will be required to provide a logical transition of unit movement plans following interdiction of the network. The discrete simulation begins the transition with the current location of each unit.

Discrete network simulations must be used to model high resolution representations of multiple units on the network. A discrete simulation will be required to locate multiple combat units in space and time and step those units through their respective movement plans. The effects of interdicting one unit, on all other units, measured in terms of collateral damage and time delays, can only be evaluated by use of a discrete simulation.

Discrete simulation algorithms are difficult to construct. To date, the interaction of several dependent attributes has not been fully identified and research work continues in that direction.

C. RECOMMENDATIONS

This report identified several research areas in which on-going work should be continued and other areas which need to be explored further.

The movement of multiple units, each modelled as a discrete packet on a complex network needs to be developed. Supporting algorithms are needed to fully model the effects of network interdiction, identify arc and node availability times, and model the route congestion in the rear areas caused by friendly units.

Work on individual attributes needs to identify those characteristics which have a significant effect on movement on the network. Examples include arc surface categories and mobility classes, what feature a node needs to represent, and what types of units will be modelled at various levels of resolution.

For the command and control model presented here, future work needs to identify each staff section which has an impact on planning at a given level of resolution, and then develop the decision algorithms for use within each processor. Identification of the exact nature of what flows on each arc needs to be accomplished for use in high resolution models.

APPENDIX A
OPERATIONAL TERMS AND DEFINITIONS

[Ref. 10]

AIRLAND BATTLE The Airland battle concept outlines an approach to military operations which realizes the full potential of US forces. Two ideas--extending the battlefield and integrating conventional nuclear, chemical, and electronic means-- are combined to describe a battlefield where the enemy is attacked to the full depth of his formations.

ATTACH The temporary placement of units or personnel in an organization. Subject to limitations imposed by the attachment order, the commander of the formation, unit, or organization receiving the attachment will exercise the same degree of command and control as he does over units and persons organic to his command. However, the responsibility for transfer and promotion of personnel will normally be retained by the parent unit, or organization.

COMMAND and CONTROL Functions performed through the arrangement of personnel, equipment, communications, facilities, and procedures which provide for direction of combat operations.

COUNTERMOBILITY OPERATIONS (ENGINEER) The construction of obstacles and reinforcement of terrain to delay, disrupt, and destroy the enemy. The primary purpose of counter-mobility operations is to slow or divert the enemy, increase time for target acquisition, and increase weapon effectiveness.

DIRECT SUPPORT The support provided by a unit or force not attached or under the command of the supported unit or force, but required to give priority to the support required by that unit or force.

FORWARD LINE OF OWN TROOPS (FLOT) A line which indicates the most forward positions of friendly forces at a specific time.

GENERAL SUPPORT Support that is given to the force as a whole and not to any particular subdivision. A mission which is frequently assigned to combat support and combat service support units. For example, a division field artillery battalion assigned to a general support mission operates under control of the division artillery while supporting the whole division.

GENERAL SUPPORT REINFORCING Artillery mission requiring the unit assigned the mission to support the force as a whole and provide reinforcing fires for another artillery unit as a second priority.

MOBILITY OPERATIONS Obstacle reduction by engineer units to reduce or negate the effects of existing or reinforcing obstacles. The objectives are to improve movement of maneuver/weapon systems and critical supplies and to construct covered and concealed routes to and from battle positions.

OBSTACLE Any natural or manmade obstruction that canalizes, delays, restricts, or divers movement of a force. The effectiveness of an obstacle is enhanced when covered by fire.

ORGANIC Assigned to and forming an essential part of a military organization; an element normally shown in the unit's table of organization and equipment (TOE).

OPERATIONAL CONTROL The authority delegated to a commander to direct forces so that he may accomplish specific missions or tasks which are usually limited by function, time, or location; and to deploy units and retain or assign tactical control of those units. It does not include administrative or logistic responsibility, discipline, internal organization, or unit training.

REINFORCING An artillery mission requiring one artillery unit to augment the fires of another artillery unit.

LIST OF REFERENCES

1. Hartman, James K., Parry, Samuel H., Schoenstadt, Arthur L., Airland Research Model, paper presented to MORs, Naval Postgraduate School, CA, June 1984
2. Schoenstadt, Arthur L., Toward an Axiomatic Generalized Value System, unpublished paper, Naval Postgraduate School, CA, June 1984
3. Needles, Christopher J., Parameterization of Terrain in Army Combat Analysis, MS Thesis, Naval Postgraduate School, CA, Mar 1976
4. Hartman, James K., Parametric Terrain and Line of Sight Modelling in the STAR Combat Model, Technical Report, NPS55-79-018, Naval Postgraduate School, CA, Aug 1979
5. Balderman, Michael A., A Division Level Communications Model for Use in the Airland Model, MS Thesis, Naval Postgraduate School, CA, June 1984
6. Wagner, Harvey M., Principles of Operations Research, 2d Edition, Prentice-Hall, New Jersey, 1975
7. Price, W. L., Graphs and Networks, An Introduction, Auerbach Publishers, England, 1971
8. Handler, Gabriel Y. and Mirichandani, Pitu B., Location on Networks, Theory and Algorithms, MIT Press, 1979
9. Luenberger, David G., Linear and Nonlinear Programming, 2d Edition, Addison-Wesley, Massachusetts, 1984
10. Skachko, P.V., Kulikov, Vi. M., Volkuv, G. K., Troop Control Through PERT Methods, (FOUO), (Translation), Foreign Broadcast Information Service, 13 Aug 81
11. RB101-999(T) Staff Officers Handbook, USACGSC, Ft. Leavenworth, KS, 1982
12. Soviet Army Operations, US Army Threat Analysis Center, April 1978
13. Organization for Combat - G-3 Worksheet, USACGSC, Ft. Leavenworth, KS, 1981

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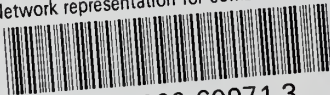
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